Structured MSc in Mathematical Sciences - January 2020 intake

6-24 January

Computing and LaTeX
Jan Groenewald (AIMS)

This course introduces students to the AIMS computing facilities and packages. The course covers an introduction to LaTeX using texmaker, followed by working through the book: http://en.wikibooks.org/wiki/LaTeX, and associated documents such as those from the American Mathematical Society.

Python
Jeff Sanders, Lebeko Poulo and Martha Kamkuemah (AIMS)

The purpose of this course is to teach students how to program, using the language Python as a vehicle. Programming is difficult, and one of the reasons is that programmers are offered no "space" in which to develop their ideas. Instead they are forced to write code, then test and modify it, iteratively, in the hope that a more accurate program result. Inevitably, it seldom does. In this course, we introduce "design space" for the specification and development of programmes. The result is that students are able to use discrete mathematics in formulating algorithms and analysing their efficiency before having to downcode to Python. Also covered: version control; unix; and shell script.

Model Theory
Charlotte Kestner (Imperial College)

This is an introductory course in model theory, which is a branch of mathematical logic. Model theory studies mathematical structures (e.g. fields, groups, graphs) paying particular attention to the language used to describe these structures. Thus from the model theoretic point of view, the integers as an additive group has very different properties to the integers as a field. This approach can be extremely fruitful. The best example of this is probably Hrushovski's proof of the Mordell Lang conjecture for function fields, a result in number theory which was proved using model theory. The course will start with some basic set theory, in particular we will cover the ultra-product construction. We will then go on to cover first order languages, and the interpretation of these languages in mathematical structures. We will prove the compactness theorem using ultraproducts and look at some applications.

This course is an abstract mathematics course. There are few prerequisites, but to enjoy the course you must be keen on abstract concepts, and willing to rigorously prove theorems using formal definitions.

Fluid Dynamics
Grae Worster (University of Cambridge)
Richard Katz (University of Oxford)

Fluids are all around us, from the air we breathe to the oceans that determine our climate and from oil that powers our industries to metals that are cast into machinery. The study of fluid dynamics requires sophisticated applications of mathematics and the ability to translate physical problems into mathematical language and back again. The course begins by building a fundamental understanding of viscous fluid flows in the context of unidirectional flows. In more general, higher dimensional flows, pressure gradients are generated within a fluid to deflect the flow around obstacles rather than the fluid being compressed in front of them, and an understanding of the coupling between momentum and mass conservation through the pressure field is key to the understanding and analysis of fluid motions. We will use simple experiments to illustrate and motivate our mathematical understanding of fluid flow. Prerequisite for the course is fluency with differential equations and vector calculus. No previous knowledge of fluid dynamics will be assumed.

Financial Mathematics
Ronnie Becker (AIMS South Africa)

The course will give an introduction to financial mathematics and will discuss the basic concepts necessary for an understanding of the subject. Topics to be covered include portfolio theory, financial Instruments, risk management, no-arbitrage pricing of assets, asset pricing in the binomial model, elements of stochastic calculus, stochastic differential equations and Monte Carlo methods for solving stochastic differential equations. Numerical methods using the computer platform Python will be employed to do calculations on financial data obtained from the internet.

27 January to 14 February

Experimental Mathematics with Sage
Eric Andriantiana, Rhodes University

This course introduces an approach to doing mathematics that is founded on experiment and inquiry. It does so in the medium of Sage, a Python-based tool for computation and experiment. Some of the problems studied come from Number Theory and some from Graph Theory. Skill with Sage will be important for many subsequent courses.

Industrial Modeling
Neville Fowkes (Western Australia)

One of the interesting features of industrial and scientific modelling is that the same phenomena occur across disciplines in slightly different guises. This is why the study of `archetypal problems' is so important. An archetypal problem should `strip away' the inessentials leaving the basic issues exposed. A list of archetypal problems is of course impossible because no agreement even between experienced `modellers' is possible, but the list would not vary by too much and largely differ because of familiarity with the context. In this course I shall present a range of archetypal problems. The appropriate mathematical procedure to use to extract results depends not only on the problem type but also on the question of interest. Also very often an approximate solution is better than an exact solution. I shall illustrate this by examining a broad range of useful techniques in the context of archetypal problems and problems arising out of continuum mechanics/industrial contexts. Techniques may include: scaling, asymptotics, singular perturbation techniques, variational methods, analytic vs numerical methods, classification of partial differential equations, Fourier methods. Phenomena may include diffusion, nonlinear vibrations, waves, shock dynamics, boundary layers, buckling, enzyme kinetics.

Introduction to Random Systems, Information Theory and Related Topics
Stéphane Ouvry (Paris-Sud University)

This course is an introduction to various random systems, probability theory, Shannon information theory and some related topics, with a special emphasis on their mathematical aspects. In particular I will present selected lectures on

- Probability calculus and the central limit theorem
- Application to random walks on a line and Brownian curves
- Notions of random numbers and pseudo random numbers
- Application to Monte Carlo sampling
- Shannon statistical entropy and information theory
- LZW compression algorithm
- Diaconis riffle shuffle: how to “randomize” a deck of cards?
- Random permutations and application to
  the statistical “curse” problem in sailing boat regattas

Networks
Phil Knight (Strathclyde University)

One cannot ignore the networks we are part of, that surround us in everyday life. There's our network of family and friends; the transport network; the banking network---it doesn't take much effort to come up with dozens of examples. Network theory aims to provide a mathematical framework for analysis of the huge networks that drive the global economy (directly or indirectly) and this course provides an introduction to the key tools and an opportunity to employ them to gain new insight into complex behaviours and structures in real-world data.

The intimate connection between matrix algebra and graph theory will be highlighted and students will use this connection to develop a practical approach to analysing networks. Python provides an ideal computational environment for large-scale simulation and analysis, in particular for identifying the key members of a network and for uncovering local and global structure that can be hidden by the scale of the data.

24 February to 13 March

Entrepreneurship case studies.
Stefan Jaehnichen (TU Berlin)

These lectures complement those given by Jonathan Marks earlier in the year. They are intended to activate students to search for possibilities to start their own companies. Entrepreneurship is seen as a way of thinking creatively rather than as the application of business rules. Examples are given of successful entrepreneurs and their companies, some of them globally known as successful start-ups, others witnessed by the lecturer or started by him. Case studies are analysed and the main factors of their success identified.

Theme and conclusion: a well-thought-out idea with some common sense is often more successful than new technology, business administration rules, and even a large amount of start-up money. This is a particularly valuable lesson in the African context (where startup funding is scarce) and for AIMS graduates (who have strong quantitative skills).

A recommended reference, taking the same approach, is “Brain versus Capital” by Günter Faltin.

Analytical Techniques in Mathematical Biology
Wilson Lamb (Strathclyde University)

Mathematical models arising in the natural sciences often involve equations which describe how the phenomena under investigation evolve in time. When time is regarded as a continuous variable such evolution equations usually take the form of differential equations. In this course, a number of mathematical techniques will be presented for analysing a range of evolution equations that arise in Biomathematics, particularly in population dynamics. The emphasis will be placed on determining qualitative features of solutions, such as the long-term behaviour. Different types of equations will be examined, but a unifying theme will be provided by developing methods from a dynamical systems point of view and using some results from functional analysis. To fix ideas, the course will begin with some simple one-dimensional models from population dynamics, such as the Malthusian and Verhulst equations. Structured population models arising in epidemiology, such as the SIS and SIR models, and multispecies models, such as the Lotka –Volterra predator-prey equations, will be considered next. The latter models result in non-linear systems of ordinary differential equations and their analysis involves higher (but still finite) dimensional dynamical systems theory. To give an indication of the need, in some problems, to work within an infinite-dimensional setting, we shall conclude by examining the notion of diffusion-driven (or Turing) instability in reaction-diffusion type partial differential equations and discuss mathematical models of pattern formation (e.g. in animal coats) that involve such equations.

Computational Algebra
Wolfram Decker and Gerhard Pfister (Kaiserslautern)

Groebner bases and Buchberger's algorithm for ideals and modules will be studied. Applications to commutative algebra, selected problems in singularity theory and algebraic geometry will be given as well as applications to electronics and engineering. The course includes an introduction to the computer algebra system SINGULAR and its programming language.

Atmospheric Modelling
Douw Steyn (University of British Columbia)
Stefano Galmarini (Institute for Environment and Sustainability, Italy)

This course will provide an introduction to general relativity. The initial emphasis will be on differential geometry before moving on to more gravitation-related aspects. Topics to be covered include Lie groups and algebras, special relativity and the Lorentz group, tensors and tensor calculus, manifolds, differential forms and integration on manifolds, curvature tensors and the geodesic equation, Lie derivatives and Killing vectors, Einstein's equations and gravitation, the Schwarzschild black hole, Penrose diagrams and cosmological solutions. The role of symmetries and invariance will be emphasised throughout.

Beyond being exposed to the basics of general relativity, the intention is for the student to acquire several important skills and learn techniques which have wide applications within mathematical and theoretical physics and to become confident in computations involving tensors, both analytic as well as using computer algebra software.

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16 March to 3 April

Mathematical Problem Solving
Nancy Neudauer (Pacific University)

In this course we shall consider a variety of elementary, but challenging, problems in different branches of pure mathematics. Investigations, comparisons of different methods of attack, literature searches, solutions and generalizations of the problems will arise in discussions in class. The objective is for students to learn, by example, different approaches to problem solving and research.

Algebraic biology

Matt Macauley (Clemson)

Mathematical biology has been transformed over the past 15 years by researchers using novel tools from discrete math and computational algebra to tackle old and new problems. For example, many systems such as gene regulatory networks have been traditionally modeled using differential equations. However, a new popular trend is to use finite dynamical systems such as Boolean networks. In this setting, the local functions and the dynamical system map can be expressed as multivariable polynomials. This opens the door to using the powerful toolbox of computational algebra to attack classic problems in systems biology. In this class, students will be introduced to this new and exciting field known as "algebraic mathematical biology".

6-24 April

Risk Management and Insurance Economics

Juerg Weber, University of Western Australia

The proposed course provides an introduction to the operation of insurance markets and the design of insurance contracts. These topics would be useful to mathematicians who are seeking a career in the actuarial sciences and the insurance industry. The course, which uses some financial mathematics and calculus, would be based on the textbook by P. Zweifel and R. Eisen, 'Insurance Economics', Berlin and Heidelberg, Springer, 2012.

Designing Algorithms
Jeff Sanders, AIMS South Africa

How efficiently can you multiply two numbers? Sort a database? Send email over an unreliable medium? Toss a coin by mobile phone? Search the web? How does quantum computing work? Bitcoin? What is a 'zero-knowledge protocol'? In diverse areas, scientists need to model new and interesting phenomena; for example in the modelling of environmental, biological, or economic systems. Typically, data are so numerous that efficient simulation —by an efficient algorithm— is essential. Being able to design algorithms, and knowing their theoretical limits, is fundamental to simulation, programming and modelling in general.

However, over the past decade the systems being modelled have changed significantly so that it is no longer sufficient to rely on the results of standard courses in Algorithm Design. Systems are distributed, use probabilistic choices, and may be quantum.

Some problems have no elegant solution (for example, there is no symmetrical algorithm to elect a leader amongst identical processors); others have no efficient solution (for a given set of integers, deciding whether or not it has a nonempty subset with sum 0); whilst yet others have no algorithm at all (deciding whether or not a program runs forever on a given input).

The purpose of this course is to provide an introduction to contemporary algorithm design, with representative examples. Against the backdrop of the traditional techniques dealing with data in one location we consider algorithms for accessing distributed data. More advanced topics will be selected according to audience interest.

4-22 May 2020

Distributed Coordination
Simukai Utete, AIMS

The course considers methods for coordinating decisions and actions of intelligent agents. In many areas, distributed agents are being deployed to solve problems in concert; for example, mobile robots can be employed in environmental monitoring. How do such systems manage failures, coordinate completion of a shared task, or decide on a common course of action? In multi-sensor networks, diverse sensor information can be combined using data fusion methods. The combination of decisions and the coordination of actions have parallels to data fusion, but significant differences too. The course will consider problems, and paradoxes, of distributed decision making and of distributed coordination. Areas covered will include decision making via voting, auctions and communication. In the practical part of the course, students will design and implement a distributed coordination mechanism to address a problem presented in the class. Understanding distributed coordination problems is important for realising systems in which individual intelligent agents can work in harmony and with high levels of autonomy.

Data Science

Bubacarr Bah and Emanuel Dufourq (AIMS South Africa)

This course provides an introduction to the recent and highly influential topics of big data, data analytics and machine learning. After setting the scene, the basic analytic techniques are introduced. Examples are studied and a balance maintained between theory and practical application (including use of Python's Scikit Learn for machine learning). Both neural networks and support vector machines are covered. A benefit of the course is an appreciation of the entrepreneurial consequences of this approach.

25 May – 12 June

Multiobjective Optimization
Surafel L Tilahun (University of Zululand)

Optimization problems and corresponding solution methods are useful in different decision making areas. Once a decision making problem is formulated as a mathematical optimization problem, a number of methods or algorithms can be used to find a solution which can be used by the decision maker. Most real problems involve multiple and usually conflicting objectives. In buying an item one always want to minimize cost and maximize quality. Hence, this course is intended to introduce students with the basic definitions and formulation of multiobjective optimization, ordering and optimality notion, preference based and non-preference based approaches, classical solution methods, evolutionary, or in general metaheuristic based solution methods along with selected case studies. The list of references is listed below.
Keywords: Multiobjective optimization, multicriteria optimization, vector optimization

List of references:

1. Ehrgott, Matthias. Multicriteria optimization. Vol. 491. Springer Science & Business Media, 2005.
2. Doumpos, Michael, and Constantin Zopounidis. Multicriteria decision aid classification methods. Vol. 73. Springer Science & Business Media, 2002.
3. Deb, Kalyanmoy. Multi-objective optimization using evolutionary algorithms. Vol. 16. John Wiley & Sons, 2001.
4. Miettinen, Kaisa. Nonlinear multiobjective optimization. Vol. 12. Springer Science & Business Media, 2012.

5. Keeney, Ralph L., and Howard Raiffa. Decisions with multiple objectives: preferences and value trade-offs. Cambridge university press, 1993.
6. Chankong, Vira, and Yacov Y. Haimes. Multiobjective decision making: theory and methodology. Courier Dover Publications, 2008.

Programming in Julia (with applications to Differential Equations).
Henri Laurie (University of Cape Town)

This course introduces you to the Julia language, a new computer language especially well-suited to scientific computing. Although what follows looks like an inflexibly designed course, this is not the case: we can extensively adapt to the students' interests. The one inflexible aspect is that a short, typeset report is required at the end of each of the three weeks on the course (topics will be set, a student can negotiate their own topic); this could be in the form a typeset pdf or (preferably) an IJulia notebook. Your mark for the course will be determined by these three project reports.

After an introduction to the basics of writing Julia code and the three main interfaces (the Julia REPL; IJulia; Juno), we look briefly at some special features of Julia. These include multiple dispatch, JIT compiling, user types, and macros. We look at Julia's package system, the packages loaded by default and how to load others. Finally, we briefly compare Julia to other languages (such as C, Python and Matlab), paying particular attention to Julia as a solution to the two-language problem and to the expression problem. We go on to the basics of solving ODE systems in Julia: formulating the problem, choosing the algorithm, computing the solution, plotting the output. The assignment for week 1 is that you report on solutions for several ODE systems. The JuliaDiffEq ODE ecosystem is very rich: a huge variety of ODE algorithms, stochastic ODEs, problems with jumps and other discontinuities, delay ODEs, boundary value problems ... and much more. We explore some of the richness in week 2; the assignment for week 2 will involve exploring in some detail one problem from one of these classes. In week 3, we focus more narrowly on one of the following: stochastic ODEs, bifurcation analysis, parameter estimation and machine learning. The assignment will be tailored to the focus of the week.

To whet your appetite, visit https://docs.julialang.org/en/v1/ and https://juliadiffeq.org/

15 June – 3 July

Numerical Mathematics

Justin Munyakazi, University of the Western Cape

Real life situations (with important exceptions of combinatorial optimization, cryptography and gene sequencing) are modelled by continuous mathematics. In the natural sciences, engineering, finance and economics, we often encounter models that employ functions of real variables. These models can be linear or nonlinear and may involve derivatives or integrals or both. In many applications, the models cannot be solved analytically. In this course we present a number of numerical techniques to construct approximate solutions. For most algorithms, the issue of efficiency and accuracy will be discussed.

Symmetry analysis of Differential Equations
Khalique Masood, North West University

Symmetry methods for solving differential equations, originally developed by Sophus Lie (1842-1899), unifies many existing ad hoc methods for constructing explicit solutions for differential equations and provide powerful new ways to find analytical solutions. Lie invented the theory of Lie groups when studying symmetries of differential equations. This theory has applications in many areas of mathematics such as algebraic topology, differential geometry, bifurcation theory, control theory, classical mechanics, relativity etc. It also has applications in physics, engineering and other mathematically-based sciences. In this course we shall learn how to find Lie point symmetries of an ordinary differential equation and then use them to find exact solutions of the equation.

October 2020

Jonathan Marks (Gordon Institute of Business Studies)

Entrepreneurship

This course introduces students to the discipline and process of entrepreneurship and new venture planning. Entrepreneurship crosses many disciplines and is a pursuit and a subject domain that is now commonly found in technical areas such as science, engineering, technology and mathematics. In this course we will introduce students to the process of originating a business idea and to creating a business model (an 'engine' for economic growth and sustainability) using a tool called the Business Model Canvas. The course will give students both the knowledge and the competence to consider entrepreneurship as part of a future career path.

Concepts and Physics Problem Solving

David Aschman, University of Cape Town

This course shows that physics describes the real world using the language of mathematics. Problem solving techniques such as changing the point of view, using different reference frames, estimating orders of magnitude, dimensional analysis, and numerical approaches will be used. Examples will be taken from physics of moving objects, electrodynamics, gravity, movement of molecules in gases, and elementary particle physics.

Students are required to read, think, discuss, engage, interact, argue, present their ideas verbally, do homework, compute and present their ideas verbally and in writing. Details of the topics covered will be available on the course page.