Structured MSc in Mathematical Sciences - August 2019 intake

11-29 November

Harmonic Analysis
Kinvi Kangi (Universite Felix Houphouet Boigny)

I'll start with studying topological groups (general case, abelian case , compact case) and some examples and after , the integration on locally compact groups and on homogeneous spaces (Haar measures which an extension of Lebesgue measures, quasi-invariant measures, Dirac measures, measure of density….) and then, the theory of representation of topological groups on a Hilbert space or on some locally convex spaces (abelian and compact cases, of torus….) and its applications. The induced representations of semi-direct product group, like some physical groups (Poincaré and Mautner groups) and its applications in quantum mechanics.
The Spherical Fourier analysis on groups will be also explored. The Fourier Analysis has many applications in solving differential equations, in signal and image process……

Quantum Mechanics
Rafael Nepomeichie (Miami University)
After reviewing elements of classical mechanics (Lagrangian, Hamiltonian, and Poisson brackets), students will apply the basic principles of quantum mechanics to the free particle, the harmonic oscillator and the spin-1/2 system. Students will then learn about higher ($s > 1/2$) spin, which provides an ideal introduction to Lie algebras and their irreducible representations. Using the notion of tensor product to describe a composite system of two spins, students will be introduced to the Clebsch-Gordan theorem. Students will gain valuable "hands on" experience by using Sage to explicitly obtain both irreducible and reducible representations of the su(2) generators.

The remainder of the course will focus on the Heisenberg quantum spin chain (a ring of N spin-1/2 spins with nearest-neighbour isotropic interactions), which is of fundamental importance in physics. Students will first study this model numerically for small values of N: they will classify the states according to energy, momentum and spin, and understand degeneracies with the help of the Clebsch-Gordan theorem. They will then rederive these results from the Bethe ansatz. Finally, after having witnessed the power of Bethe ansatz, students will discover the source of its "magic": they will be introduced to the famous Yang-Baxter equation, and work out the algebraic Bethe ansatz solution of the Heisenberg chain. Computer techniques such as the Newton-Raphson method and recursive programming will also be reviewed and used.

Differential Equations
Fernando Pestana da Costa (Aberta University)

The goal of this course is to be an introduction to the theory and applications of Ordinary Differential Equations, with emphasis in methods of qualitative theory. It starts by revisiting basic techniques to solve ODEs: separable equations, integrating factors, and changes of variables. After these preliminaries the general theorem of Picard-Lindelof for existence and uniqueness of solutions to initial value problems, as well as results on dependence of solutions on the initial data and parameters are studied. These preliminary general results are followed by the main part of the course, viz. a study of important tools from qualitative theory. These consist in the introduction of the main concepts (phase space, flows, critical points, orbits, conservative systems, first integrals, phase portraits, etc.), notions and results about stability of solutions, Lyapunov functions, limit sets, and a study of planar autonomous systems. The final part of the course deals with linear systems (including the computation of the matrix exponential) and the study of the linearization method for nonlinear systems (including the notion of conjugation, and the statement ---without proofs--- and use of the Hartman-Grobman and Hadamard-Perron theorems.) All topics are motivated and exemplified by adequate examples. The discussion and resolution of exercises and problems illustrating the notions, results, and methods studied is an integral part of the course.

2-20 December

Knot theory
Bruce Bartlett (Stellenbosch University)

A mathematical "knot" is an embedding of the circle into three-dimensional space, in other words it is a "closed loop" in space. Two knots are equivalent if one can be continuously deformed into the other, and we are interested in classifying knots up to equivalence. This course serves as an introduction to low-dimensional topology. We will draw a lot of pictures but still present rigorous mathematical proofs. Topics to be covered: Knots and their diagrams, Reidemeister's Theorem, Linking number, Kauffman bracket, Jones polynomial.

Notes: Justin Roberts, Knots Knotes (available online).

Signal Processing with Python
Paul Taylor (National Institute of Mental Health, Bethesda, USA)
The goal of this course is to cover an introductory review of signal processing methods. We will investigate both analytic formulations and computational applications. Signal analysis is routinely performed to enhance parameters in detecting and identifying specific regions of interest. The processing methods encompass signal detection, noise reduction, and image transformation. A familiarity with signals is a broadly applicable and increasingly useful skill set due to the continued digitalization of the world. Whether making medical diagnoses, working in the field of communications, or simply gathering data from electrical equipment, signal processing is important. Mathematically, this field also represents an interesting intersection of (linear) algebra, statistics and calculus. Obviously, computers are closely tied to analysis, and the course emphasizes their role in analysis. Topics include: Fourier series/transforms, spectral analyses, impulse response functions, convolution, difference equations, filtering, smoothing, correlation/covariance, and modelling techniques.

9-13 December

Data Analytics
Pete Grinrod (Oxford University)
Jeff Sanders (AIMS South Africa)

This course provides an introduction to the recent but influential subject of Big Data and Data Analytics. After setting the scene, the basic analytic techniques are introduced. Many examples are discussed to demonstrate the benefits of this approach to modelling big data sets; use of tools is also included. These methods and tools are applied to practical problems, to the extent that one benefit of the course will be an appreciation of the entrepreneurial consequences of this approach.

Reference: P. Grindrod, “Mathematical Underpinnings of Analytics”, Oxford University Press.

6-24 January

Mathematics in Industry Study Group
Workshop

7-25 January

Model Theory
Charlotte Kestner (Imperial College)

This is an introductory course in model theory, which is a branch of mathematical logic. Model theory studies mathematical structures (e.g. fields, groups, graphs) paying particular attention to the language used to describe these structures. Thus from the model theoretic point of view, the integers as an additive group has very different properties to the integers as a field. This approach can be extremely fruitful. The best example of this is probably Hrushovski’s proof of the Mordell Lang conjecture for function fields, a result in number theory which was proved using model theory. The course will start with some basic set theory, in particular we will cover the ultra-product construction. We will then go on to cover first order languages, and the interpretation of these languages in mathematical structures. We will prove the compactness theorem using ultraproducts and look at some applications.

This course is an abstract mathematics course. There are few prerequisites, but to enjoy the course you must be keen on abstract concepts, and willing to rigorously prove theorems using formal definitions.

Fluid Dynamics
Grae Worster (University of Cambridge)
Richard Katz (University of Oxford)

Fluids are all around us, from the air we breathe to the oceans that determine our climate and from oil that powers our industries to metals that are cast into machinery. The study of fluid dynamics requires sophisticated applications of mathematics and the ability to translate physical problems into mathematical language and back again. The course begins by building a fundamental understanding of viscous fluid flows in the context of unidirectional flows. In more general, higher dimensional flows, pressure gradients are generated within a fluid to deflect the flow around obstacles rather than the fluid being compressed in front of them, and an understanding of the coupling between momentum and mass conservation through the pressure field is key to the understanding and analysis of fluid motions. We will use simple experiments to illustrate and motivate our mathematical understanding of fluid flow. Prerequisite for the course is fluency with differential equations and vector calculus. No previous knowledge of fluid dynamics will be assumed.

Financial Mathematics
Ronnie Becker (AIMS South Africa)
Hans-Georg Zimmerman

The course will give an introduction to financial mathematics and will discuss the basic concepts necessary for an understanding of the subject. Topics to be covered include portfolio theory, financial Instruments, risk management, no-arbitrage pricing of assets, asset pricing in the binomial model, elements of stochastic calculus, stochastic differential equations and Monte Carlo methods for solving stochastic differential equations. Numerical methods using the computer platform Python will be employed to do calculations on financial data obtained from the internet.

27 January to 14 February

Industrial Modeling
Neville Fowkes (Western Australia)

One of the interesting features of industrial and scientific modelling is that the same phenomena occur across disciplines in slightly different guises. This is why the study of `archetypal problems' is so important. An archetypal problem should `strip away' the inessentials leaving the basic issues exposed. A list of archetypal problems is of course impossible because no agreement even between experienced `modellers' is possible, but the list would not vary by too much and largely differ because of familiarity with the context. In this course I shall present a range of archetypal problems. The appropriate mathematical procedure to use to extract results depends not only on the problem type but also on the question of interest. Also very often an approximate solution is better than an exact solution. I shall illustrate this by examining a broad range of useful techniques in the context of archetypal problems and problems arising out of continuum mechanics/industrial contexts. Techniques may include: scaling, asymptotics, singular perturbation techniques, variational methods, analytic vs numerical methods, classification of partial differential equations, Fourier methods. Phenomena may include diffusion, nonlinear vibrations, waves, shock dynamics, boundary layers, buckling, enzyme kinetics.

Introduction to Random Systems, Information Theory and Related Topics
Stéphane Ouvry (Paris-Sud University)

This course is an introduction to various random systems, probability theory, Shannon information theory and some related topics, with a special emphasis on their mathematical aspects. In particular I will present selected lectures on

- Probability calculus and the central limit theorem
- Application to random walks on a line and Brownian curves
- Notions of random numbers and pseudo random numbers
- Application to Monte Carlo sampling
- Shannon statistical entropy and information theory
- LZW compression algorithm
- Diaconis riffle shuffle: how to “randomize” a deck of cards?
- Random permutations and application to
  the statistical “curse” problem in sailing boat regattas

Networks
Phil Knight (Strathclyde University)

One cannot ignore the networks we are part of, that surround us in everyday life. There's our network of family and friends; the transport network; the banking network---it doesn't take much effort to come up with dozens of examples. Network theory aims to provide a mathematical framework for analysis of the huge networks that drive the global economy (directly or indirectly) and this course provides an introduction to the key tools and an opportunity to employ them to gain new insight into complex behaviours and structures in real-world data.

The intimate connection between matrix algebra and graph theory will be highlighted and students will use this connection to develop a practical approach to analysing networks. Python provides an ideal computational environment for large-scale simulation and analysis, in particular for identifying the key members of a network and for uncovering local and global structure that can be hidden by the scale of the data.

24 February to 28 March

Entrepreneurship case studies.
Stefan Jaehnichen (TU Berlin)

These lectures complement those given by Jonathan Marks earlier in the year. They are intended to activate students to search for possibilities to start their own companies. Entrepreneurship is seen as a way of thinking creatively rather than as the application of business rules. Examples are given of successful entrepreneurs and their companies, some of them globally known as successful start-ups, others witnessed by the lecturer or started by him. Case studies are analysed and the main factors of their success identified.

Theme and conclusion: a well-thought-out idea with some common sense is often more successful than new technology, business administration rules, and even a large amount of start-up money. This is a particularly valuable lesson in the African context (where startup funding is scarce) and for AIMS graduates (who have strong quantitative skills).

A recommended reference, taking the same approach, is “Brain versus Capital” by Günter Faltin.

24 February to 13 March

Analytical Techniques in Mathematical Biology
Wilson Lamb (Strathclyde University)

Mathematical models arising in the natural sciences often involve equations which describe how the phenomena under investigation evolve in time. When time is regarded as a continuous variable such evolution equations usually take the form of differential equations. In this course, a number of mathematical techniques will be presented for analysing a range of evolution equations that arise in Biomathematics, particularly in population dynamics. The emphasis will be placed on determining qualitative features of solutions, such as the long-term behaviour. Different types of equations will be examined, but a unifying theme will be provided by developing methods from a dynamical systems point of view and using some results from functional analysis. To fix ideas, the course will begin with some simple one-dimensional models from population dynamics, such as the Malthusian and Verhulst equations. Structured population models arising in epidemiology, such as the SIS and SIR models, and multispecies models, such as the Lotka –Volterra predator-prey equations, will be considered next. The latter models result in non-linear systems of ordinary differential equations and their analysis involves higher (but still finite) dimensional dynamical systems theory. To give an indication of the need, in some problems, to work within an infinite-dimensional setting, we shall conclude by examining the notion of diffusion-driven (or Turing) instability in reaction-diffusion type partial differential equations and discuss mathematical models of pattern formation (e.g. in animal coats) that involve such equations.

Computational Algebra
Wolfram Decker and Gerhard Pfister (Kaiserslautern)

Groebner bases and Buchberger's algorithm for ideals and modules will be studied. Applications to commutative algebra, selected problems in singularity theory and algebraic geometry will be given as well as applications to electronics and engineering. The course includes an introduction to the computer algebra system SINGULAR and its programming language.

Atmospheric Modelling
Douw Steyn (University of British Columbia)
Stefano Galmarini (Institute for Environment and Sustainability, Italy)

This course will provide an introduction to general relativity. The initial emphasis will be on differential geometry before moving on to more gravitation-related aspects. Topics to be covered include Lie groups and algebras, special relativity and the Lorentz group, tensors and tensor calculus, manifolds, differential forms and integration on manifolds, curvature tensors and the geodesic equation, Lie derivatives and Killing vectors, Einstein's equations and gravitation, the Schwarzschild black hole, Penrose diagrams and cosmological solutions. The role of symmetries and invariance will be emphasised throughout.

Beyond being exposed to the basics of general relativity, the intention is for the student to acquire several important skills and learn techniques which have wide applications within mathematical and theoretical physics and to become confident in computations involving tensors, both analytic as well as using computer algebra software.

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16 March to 3 April

Solitons
Patrick Dorey (Durham University)

Solitons were first identified in 1834 - they are exceptionally long-lasting solutions to certain non-linear partial differential equations, with many intriguing properties and widespread applications in mathematics and physics. This course will explore some aspects of this still-developing story.

Topics covered will include:
* Illustrations of solitons via computer animations and practical demonstrations (this will introduce the KdV and sine-Gordon equations, two key examples for soliton theory)
* Waves, dispersion and dissipation
* Travelling waves
* Numerical simulation of soliton equations
* A quick sketch of Lagrangian mechanics (first for particles and then for field theory)
* Topological lumps and the Bogomolnyi argument; sine-Gordon and phi4 examples (phi4 is a further equation with soliton-like properties)
* Conservation laws; extra conservation laws for the KdV equation
* Backlund transformations as a tool to generate exact solutions of soliton equations

Algebraic biology

Matt Macauley (Clemson)

Mathematical biology has been transformed over the past 15 years by researchers using novel tools from discrete math and computational algebra to tackle old and new problems. For example, many systems such as gene regulatory networks have been traditionally modeled using differential equations. However, a new popular trend is to use finite dynamical systems such as Boolean networks. In this setting, the local functions and the dynamical system map can be expressed as multivariable polynomials. This opens the door to using the powerful toolbox of computational algebra to attack classic problems in systems biology. In this class, students will be introduced to this new and exciting field known as "algebraic mathematical biology".

Elliptic curves, elliptic functions, and cryptography
Jorg Zintl (TU Kaiserslautern)

Elliptic curves show up in many different areas of mathematics. Therefore, they provide an ideal starting point for expeditions into a variety of fields, as for example algebraic geometry, complex analysis, differential geometry, topology and even cryptography. The course is intended as an interdisciplinary class. Its focus lies on comparing geometric, analytic and algebraic methods, and some aspects of elliptic curve cryptography.

At first, elliptic curves will be introduced as objects of projective geometry. Here, the geometry of lines in the plane leads to algebraic structures (groups), and to algebraic geometry. In the case of elliptic curves, the powerful but abstract methods of this geometry can be made very explicit. All definitions and constructions of this course aim to be on a very elementary level and as constructive as possible. By translating geometry into algebra, the studies are no longer confined to the field of complex numbers. In particular, elliptic curves over finite fields have important applications in modern day cryptography. Some of these aspects will be addressed in the course. Finally, a complex elliptic curve relates to a certain complex torus. Complex tori are well-known objects in topology and differential geometry. This leads to the study of certain periodic meromorphic functions in the context of complex analysis, and to the theory of elliptic functions.