**Structured MSc in Mathematical Sciences - August 2018 intake**

12-30 November

**Algebraic Methods**
**Karin-Therese Howell (Stellenbosch University)**

The course will have an Algebra and a Linear Algebra component. Under Algebra we will discuss functions, relations, partitions, groups, morphisms, quotients, the Isomorphism Theorems for groups and direct products. As part of the Linear Algebra component we will discuss linear spaces, linear functionals and operators, matrices, change of basis, eigenvalues and eigenvectors and the normal form.

**Differential Equations**
**Fernando Pestana da Costa (Aberta University)**

The goal of this course is to be an introduction to the theory and applications of Ordinary Differential Equations, with emphasis in methods of qualitative theory. It starts by revisiting basic techniques to solve ODEs: separable equations, integrating factors, and changes of variables. After these preliminaries the general theorem of Picard-Lindelof for existence and uniqueness of solutions to initial value problems, as well as results on dependence of solutions on the initial data and parameters are studied. These preliminary general results are followed by the main part of the course, viz. a study of important tools from qualitative theory. These consist in the introduction of the main concepts (phase space, flows, critical points, orbits, conservative systems, first integrals, phase portraits, etc.), notions and results about stability of solutions, Lyapunov functions, limit sets, and a study of planar autonomous systems. The final part of the course deals with linear systems (including the computation of the matrix exponential) and the study of the linearization method for nonlinear systems (including the notion of conjugation, and the statement ---without proofs--- and use of the Hartman-Grobman and Hadamard-Perron theorems.) All topics are motivated and exemplified by adequate examples. The discussion and resolution of exercises and problems illustrating the notions, results, and methods studied is an integral part of the course.

**3-21 December**

**Model Theory**
**Gareth Boxall (Stellenbosch University)**

Model theory is a branch of mathematical logic which has applications in several other areas of mathematics including algebraic geometry, number theory and combinatorics. This course will introduce model theory from the beginning, including some preliminaries on ordinals and cardinals. The focus will then be on reviewing several interesting applications. In particular, we shall devote significant time to non-standard analysis.

**Data Analytics**
**Pete Grinrod (Oxford University)**
**Jeff Sanders (AIMS South Africa)**

This course provides an introduction to the recent but influential subject of Big Data and Data Analytics. After setting the scene, the basic analytic techniques are introduced. Many examples are discussed to demonstrate the benefits of this approach to modelling big data sets; use of tools is also included. These methods and tools are applied to practical problems, to the extent that one benefit of the course will be an appreciation of the entrepreneurial consequences of this approach.

Reference: P. Grindrod, "Mathematical Underpinnings of Analytics", Oxford University Press.

**Topics in Scientific Computing with Applications (to be confirmed)**
**Michael Winckler and Christian Mittlestaedt (Heidelberg University)**

**9-18 January**

**Mathematics in Industry Study Group**
Workshop

**7-25 January**

**Algebraic Number Theory**
**Rene Schoof (Leiden) and Peter Stevenhagen (Roma Tor Vergata)**

The course deals with the arithmetic of number rings. Such rings arose in the study of Diophantine equations, and are currently used in the "number field sieve" algorithm for factoring large integers into prime factors.

Basic knowledge of linear algebra and some familiarity with groups, rings and ideals are prerequisites for this course. The concepts of rings of integers, unit groups, ideal groups, ideal class groups and lattices in Euclidean space will be treated in detail.

We will emphasize practical and explicit computations. The students will be introduced to the open source software package Pari-gp.

**Fluid Dynamics**
**Herbert Huppert (University of Cambridge)**
**Richard Katz (University of Oxford)**

Fluids are all around us, from the air we breathe to the oceans that determine our climate and from oil that powers our industries to metals that are cast into machinery. The study of fluid dynamics requires sophisticated applications of mathematics and the ability to translate physical problems into mathematical language and back again. The course begins by building a fundamental understanding of viscous fluid flows in the context of unidirectional flows. In more general, higher dimensional flows, pressure gradients are generated within a fluid to deflect the flow around obstacles rather than the fluid being compressed in front of them, and an understanding of the coupling between momentum and mass conservation through the pressure field is key to the understanding and analysis of fluid motions. We will use simple experiments to illustrate and motivate our mathematical understanding of fluid flow. Prerequisite for the course is fluency with differential equations and vector calculus. No previous knowledge of fluid dynamics will be assumed.

**Financial Mathematics**
**Ronnie Becker (AIMS South Africa)**

The course will give an introduction to financial mathematics and will discuss the basic concepts necessary for an understanding of the subject. Topics to be covered include portfolio theory, financial Instruments, risk management, no-arbitrage pricing of assets, asset pricing in the binomial model, elements of stochastic calculus, stochastic differential equations and Monte Carlo methods for solving stochastic differential equations. Numerical methods using the computer platform Python will be employed to do calculations on financial data obtained from the internet.

**28 January to 15 February**

**Industrial Modeling**
**Neville Fowkes (Western Australia)**

One of the interesting features of industrial and scientific modelling is that the same phenomena occur across disciplines in slightly different guises. This is why the study of `archetypal problems' is so important. An archetypal problem should `strip away' the inessentials leaving the basic issues exposed. A list of archetypal problems is of course impossible because no agreement even between experienced `modellers' is possible, but the list would not vary by too much and largely differ because of familiarity with the context. In this course I shall present a range of archetypal problems. The appropriate mathematical procedure to use to extract results depends not only on the problem type but also on the question of interest. Also very often an approximate solution is better than an exact solution. I shall illustrate this by examining a broad range of useful techniques in the context of archetypal problems and problems arising out of continuum mechanics/industrial contexts. Techniques may include: scaling, asymptotics, singular perturbation techniques, variational methods, analytic vs numerical methods, classification of partial differential equations, Fourier methods. Phenomena may include diffusion, nonlinear vibrations, waves, shock dynamics, boundary layers, buckling, enzyme kinetics.

**Introduction to Random Systems, Information Theory and Related Topics**
**Stéphane Ouvry (Paris-Sud University)**

This course is an introduction to various random systems, probability theory, Shannon information theory and some related topics, with a special emphasis on their mathematical aspects. In particular I will present selected lectures on

- Probability calculus and the central limit theorem
- Application to random walks on a line and Brownian curves
- Notions of random numbers and pseudo random numbers
- Application to Monte Carlo sampling
- Shannon statistical entropy and information theory
- LZW compression algorithm
- Diaconis riffle shuffle: how to “randomize” a deck of cards?
- Random permutations and application to
  the statistical “curse” problem in sailing boat regattas

**Quantum Mechanics**
**Herbert Weigel (Stellenbosch University)**

Review failures of deterministic description in classical mechanics and electrodynamics; introduction to probabilistic concepts and wave-mechanics and motivation of Schroedinger equation; harmonic oscillator and piece-wise constant potentials in one dimension; Schroedinger equation in three dimensions; spherically symmetric potentials and degeneracy; orbital angular momentum: eigenvalues and spherical harmonic functions; Coulomb problem and hydrogen atom; angular momentum and spin; time-independent perturbation theory and application to the hydrogen atom; electrons in periodic crystal potentials (one space dimension).

**25 February to 1 March**

**Entrepreneurship case studies.**
**Stefan Jaehnichen (TU Berlin)**

These lectures complement those given by Jonathan Marks earlier in the year. They are intended to activate students to search for possibilities to start their own companies. Entrepreneurship is seen as a way of thinking creatively rather than as the application of business rules. Examples are given of successful entrepreneurs and their companies, some of them globally known as successful start-ups, others witnessed by the lecturer or started by him. Case studies are analysed and the main factors of their success identified.

Theme and conclusion: a well-thought-out idea with some common sense is often more successful than new technology, business administration rules, and even a large amount of start-up money. This is a particularly valuable lesson in the African context (where startup funding is scarce) and for AIMS graduates (who have strong quantitative skills).

A recommended reference, taking the same approach, is “Brain versus Capital” by Günter Faltin.

**25 February to 15 March**

**Analytical Techniques in Mathematical Biology**
**Wilson Lamb (Strathclyde University)**

Mathematical models arising in the natural sciences often involve equations which describe how the phenomena under investigation evolve in time. When time is regarded as a continuous variable such evolution equations usually take the form of differential equations. In this course, a number of mathematical techniques will be presented for analysing a range of evolution equations that arise in Biomathematics, particularly in population dynamics. The emphasis will be placed on determining qualitative features of solutions, such as the long-term behaviour. Different types of equations will be examined, but a unifying theme will be provided by developing methods from a dynamical systems point of view and using some results from functional analysis. To fix ideas, the course will begin with some simple one-dimensional models from population dynamics, such as the Malthusian and Verhulst equations. Structured population models arising in epidemiology, such as the SIS and SIR models, and multispecies models, such as the Lotka –Volterra predator-prey equations, will be considered next. The latter models result in non-linear systems of ordinary differential equations and their analysis involves higher (but still finite) dimensional dynamical systems theory. To give an indication of the need, in some problems, to work within an infinite-dimensional setting, we shall conclude by examining the notion of diffusion-driven (or Turing) instability in reaction-diffusion type partial differential equations and discuss mathematical models of pattern formation (e.g. in animal coats) that involve such equations.

**Computational Algebra**
**Wolfram Decker and Gerhard Pfister (Kaiserslautern)**

Groebner bases and Buchberger's algorithm for ideals and modules will be studied. Applications to commutative algebra, selected problems in singularity theory and algebraic geometry will be given as well as applications to electronics and engineering. The course includes an introduction to the computer algebra system SINGULAR and its programming language.

**General Relativity**
**Costas Zoubos (University of Pretoria)**

This course will provide an introduction to general relativity. The initial emphasis will be on differential geometry before moving on to more gravitation-related aspects. Topics to be covered include Lie groups and algebras, special relativity and the Lorentz group, tensors and tensor calculus, manifolds, differential forms and integration on manifolds, curvature tensors and the geodesic equation, Lie derivatives and Killing vectors, Einstein's equations and gravitation, the Schwarzschild black hole, Penrose diagrams and cosmological solutions. The role of symmetries and invariance will be emphasised throughout.

Beyond being exposed to the basics of general relativity, the intention is for the student to acquire several important skills and learn techniques which have wide applications within mathematical and theoretical physics and to become confident in computations involving tensors, both analytic as well as using computer algebra software.

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**18 March to 5 April**

**Solitons**
**Patrick Dorey (Durham University)**

Solitons were first identified in 1834 - they are exceptionally long-lasting solutions to certain non-linear partial differential equations, with many intriguing properties and widespread applications in mathematics and physics. This course will explore some aspects of this still-developing story.

Topics covered will include:
* Illustrations of solitons via computer animations and practical demonstrations (this will introduce the KdV and sine-Gordon equations, two key examples for soliton theory)
* Waves, dispersion and dissipation
* Travelling waves
* Numerical simulation of soliton equations
* A quick sketch of Lagrangian mechanics (first for particles and then for field theory)
* Topological lumps and the Bogomolnyi argument; sine-Gordon and phi4 examples (phi4 is a further equation with soliton-like properties)
* Conservation laws; extra conservation laws for the KdV equation
* Backlund transformations as a tool to generate exact solutions of soliton equations

**Introduction to Quantum Field Theory**

**Algebraic biology**

**Elena Dimitrova (Clemson)**

Mathematical biology has been transformed over the past 15 years by researchers using novel tools from discrete math and computational algebra to tackle old and new problems. For example, many systems such as gene regulatory networks have been traditionally modeled using differential equations. However, a new popular trend is to use finite dynamical systems such as Boolean networks. In this setting, the local functions and the dynamical system map can be expressed as multivariable polynomials. This opens the door to using the powerful toolbox of computational algebra to attack classic problems in systems biology. In this class, students will be introduced to this new and exciting field known as "algebraic mathematical biology.".