

# ISOP2023

## Integrable Systems and Orthogonal Polynomials

*Numerical and Analytical perspectives*

11–15 April 2023

African Institute for Mathematical Sciences  
6 Melrose Road  
Muizenberg Cape Town  
South Africa

### Abstracts



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# Plenary Talks

## *Two variable Freud orthogonal polynomials and matrix Painlevé-type difference equations*

**Cleonice Bracciali** (Universidade Estadual Paulista, Brazil)

09:00–10:00 Tuesday 11th April Main Lecture Hall

We present bivariate orthogonal polynomials associated with Freud weight functions depending on real parameters. We analyze relations between the matrix coefficients of the three term relations for the orthonormal polynomials as well as the coefficients of the structure relations satisfied by these bivariate semiclassical orthogonal polynomials. A matrix differential-difference equation for the bivariate orthogonal polynomials is deduced. The extension of the Painlevé equation for the coefficients of the three term relations to the bivariate case and a two dimensional version of the Langmuir lattice are obtained. Joint work with G.S. Costa and T.E. Pérez

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## *Rational solutions of Painlevé equations*

**Peter Clarkson** (University of Kent, UK)

11:00–12:00 Thursday 13th April Online

In this talk I shall give a review of rational solutions of Painlevé equations. Although the general solutions of the six Painlevé equations are transcendental, all except the first Painlevé equation possess rational solutions for certain values of the parameters. These solutions are usually expressed in terms of logarithmic derivatives of special polynomials that are Wronskians, often of classical orthogonal polynomials such as Hermite and Laguerre polynomials. It is also known that the roots of these special polynomials are highly symmetric in the complex plane. The polynomials arise in applications such as random matrix theory, vortex dynamics, in supersymmetric quantum mechanics, as coefficients of recurrence relations for semi-classical orthogonal polynomials and are examples of exceptional orthogonal polynomials.

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## *Polynomials and partitions*

**Clare Dunning** (University of Kent, UK)

09:00-10:00 Friday 14th April Main Lecture Hall

In this talk I discuss one family of rational solutions of the fifth Painlevé equation expressed in terms of determinants of Laguerre polynomials. I will also show that the partitions that label the allowed sets of Laguerre polynomials play an intriguing role in properties of the solutions. The talk is based on joint work with Peter Clarkson (University of Kent, UK).

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## *On properties of Askey-Wilson polynomials system and its $r^h$ -Associated*

**Maurice Kenfack Nangho** (University of Dschang, Cameroon)

09:00–10:00 Saturday 15th April Online

Askey–Wilson polynomials are a family of orthogonal polynomials introduced by Askey and Wilson (1985) as  $q$ -analogs of the Wilson polynomials. They include the other classical orthogonal polynomials in one variable as special or limiting cases. In this talk, I develop two analogs of power basis in the  $q$ -quadratic variable, show how they can be used to find series solutions to holonomic equations involving Askey’s operator, and derive two independent solutions to Sturm-Liouville type equations of Askey-Wilson polynomials (for the special values  $b = aq^{1/2}$  and  $d = cq^{1/2}$ ); characterize these polynomials by means of Pearson type equation, two structure relations, orthogonality of the derivatives and Riccati equation (These results include proof of Mourad Ismail’s Conjecture). I then use this characterization to find by general theory solutions to the indeterminate moment problem associated with Askey-Wilson polynomials when one of the parameters goes to infinity. Finally, I establish functional equation and structure relation of the  $r^h$ -associated Askey-Wilson polynomials and prove equivalence between them and Riccati equation.

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## *Special function positive solutions of the first discrete Painlevé hierarchy*

**Ana Loureiro** (University of Kent, UK)

9:00–10:00    Wednesday 12th April    Main Lecture Hall

The recurrence coefficients in the three-term recurrence relation of a polynomial sequence orthogonal with respect to a quartic, a sextic or higher order Freud weights are a solution of a fourth order discrete equation which is a member of the first discrete Painlevé hierarchy. They also satisfy a coupled system of second-order, nonlinear differential equations. Such orthogonality weights also arise in the context of Hermitian matrix models and random symmetric matrix ensembles. In this talk I will report on the behaviour of such special function solutions and explain how the study may inform on the study of recurrence coefficients associated with higher order Freud weights. The emphasis will be on their asymptotic periodic properties. Collaborators: Peter Clarkson (University of Kent, UK) and Kerstin Jordaan (University of South Africa).

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# Invited talks

## *Orthogonal polynomials and the associated Jacobi operator*

**Christian Berg** (University of Copenhagen, Denmark)

11:00–11:45 Tuesday 11th April Main Lecture Hall

To a sequence of orthonormal polynomials  $p_n$  on the real line is associated a Jacobi operator  $(T, D(T))$ , i.e., the operator in  $\ell^2$  defined as the closure of the Jacobi matrix acting on the subspace of complex sequences with only finitely many non-zero terms. It is well-known that it is symmetric with deficiency indices either  $(0,0)$  or  $(1,1)$ . The two cases correspond to the moment problem behind being either determinate or indeterminate, i.e., there is exactly one orthogonality measure or there are several and then infinitely many orthogonality measures for  $p_n$ . In the determinate case  $(T, D(T))$  is self-adjoint, but not in the indeterminate case, where it has infinitely many self-adjoint extensions. We shall focus on this case, and for a complex number  $z$  we let  $p_z, q_z$  denote the sequences  $(p_n(z))$  and  $(q_n(z))$ , where  $q_n$  denote the polynomials of the second kind. These sequence are known to be square summable. It is known that  $p_z, q_z \notin D(T)$  for all  $z \in \mathbb{C}$ . We determine whether linear combinations of  $p_u, p_v, q_u, q_v$  for  $u, v \in \mathbb{C}$  belong to  $D(T)$  or to the domain of the self-adjoint extensions of  $T$  in  $\ell^2$ . The results depend on the four Nevanlinna functions of two variables associated with the moment problem. We also show that  $D(T)$  is the common range of an explicitly constructed family of bounded operators on  $\ell^2$ . The talk is based on recent joint work with Ryszard Szwarc, Wrocław, see arXiv:2301.00586.

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## *Quantum dynamical semigroups and its generators*

**Christian Budde** (University of the Free State, RSA)

16:00–16:30 Friday 14th April Online

The dynamics of a finite closed quantum system is conventionally represented by a one-parameter group of unitary transformations in Hubert space. This formalism makes it difficult to describe irreversible processes like the decay of unstable particles, approach to thermodynamic equilibrium and measurement processes. The axioms for a dynamical semigroup as given by Ingarden and Kossakowski. In particular, a dynamical semigroup is a one-parameter family of operators on a von Neumann algebra satisfying certain properties. The purpose of this talk is to derive an explicit form for the generators of a dynamical semigroup.

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## *Uniform approximation and polynomial preimages*

**Jacob Christiansen** (Lund University, Sweden)

14:45–15:30 Tuesday 11th April Main Lecture Hall

Let  $E \subset \mathbb{C}$  be an infinite compact set of logarithmic capacity 1 and denote by  $T_n$  the Chebyshev (or minimax) polynomials of  $E$ , that is, the monic degree  $n$  polynomials which minimize the sup-norm on  $E$ . A classical result of Szegő states that  $\|T_n\|_E \geq 1$  for all  $n$ , a lower bound that doubles when  $E \subset \mathbb{R}$ . More recently, Totik has proved that for real subsets,  $\|T_n\|_E \rightarrow 2$  if and only if  $E$  is an interval. We shall pose the question if there are more subsets of  $\mathbb{C}$  for which this limit is 2 and show the answer is in the affirmative for certain polynomial preimages. Interestingly, our proof relies on properties of the (orthogonal) Jacobi polynomials due to Bernstein. Related open questions will also be discussed. The talk is based on joint work with B. Eichinger (TU Wien) and O. Rubin (Lund).

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## *Nonassociative Algebras in Integrable Systems*

**Mafoya Landry Dassoundo** (AIMS, South Africa)

16:30–17:00 Friday 14th April Main Lecture Hall

Fundamentals on some well studied nonassociative algebras as well as related consequences will be given. Remarkable observation by S. Svinolupov linking some multiple component nonlinear integrable equations to certain nonassociative algebras will be presented and some consequences will be presented.

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## *Reduction of the Pfaff lattice via the map Toda-Pfaff*

**Marta Dell'Atti** (University of Portsmouth, UK)

16:00–16:30    Wednesday 12th April    Main Lecture Hall

The discrete integrable structure concealed by symmetric matrix ensembles is the Pfaff lattice, built on a semi-infinite moment matrix defined for a skew-symmetric weight. The dependence on the infinitely many times is encoded at the level of the weight, and gives rise to an integrable hierarchy expressed as the collection of infinitely many Lax equations. The leading order of the continuum limit of the variables populating the lattice can be recast in the form of an integrable hydrodynamic chain for every time, producing a hierarchy of hydrodynamic chains. With all the times set to zero, the eigenfunctions for the Pfaff lattice are semi-infinite skew-orthogonal polynomials and can be mapped onto orthogonal polynomials, eigenfunctions for the Toda lattice. The Pfaff lattice structure is completely determined and it offers valuable insight for the form of the field variables with nonzero times.

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## *On the quasi-Painlevé equations*

**Galina Filipuk** (University of Warsaw, Poland)

14:00–14:45    Tuesday 11th April    Online

In this talk I describe the quasi-Painlevé property of a system of ordinary differential equations in terms of a global Hamiltonian structure on an analogue of Okamoto's space of initial conditions for the Painlevé equations. In the quasi-Painlevé case, the Hamiltonian structure is with respect to a two-form which is allowed to have certain zeroes on the surfaces forming the space of initial conditions, as opposed to holomorphic symplectic forms in the case of the Painlevé equations. The talk is based on a joint work with A. Stokes (University of Tokyo, Japan)

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## *Classification of rational solutions to higher order Painlevé systems*

**David Gómez-Ullate** (IE University, Spain)

11:45–12:30    Friday 14th April    Online

We generalize the known Wronskian representation of rational solutions to Painlevé PIV (generalized Hermite, Okamoto) to higher order Painlevé systems of type  $A_{2n}$ . We provide a characterization result and a full classification based on combinatorial objects. The result is a full classification and an explicit representation of all rational solutions of higher order Painlevé systems (Noumi-Yamada) related to odd-cyclic dressing chains. The Weyl symmetry group action is also described in the new representation, thus bridging the gap with the standard approach to this problem.

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## *Symmetries and some recent integrability applications*

**Sameerah Jamal** (University of the Witwatersrand, RSA)

16:30–17:00    Tuesday 11th April    Main Lecture Hall

In this talk, we discuss the one-parameter Lie group transformations that leave differential equations invariant. These transformations may be linked to several important concepts, such as conservation laws and recursion operators. We showcase how these concepts may be used to determine invariant solutions for systems of equations that possess unknown potential functions. In a second application, we consider hierarchies of partial differential equations that admit recursion operators.

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## *On the zeros of Meixner polynomials*

**Alta Jooste** (University of Pretoria, RSA)

11:45–12:30 Saturday 15th April Main Lecture Hall

In his 1934 paper, Josef Meixner classifies five classes of orthogonal polynomials,  $p_n$ , with generating function  $f(t)e^{xu(t)} = \sum_{n=0}^{\infty} c_n p_n(x)t^n$ , which include the so-called Meixner polynomials. The Meixner polynomials lie on the  ${}_2F_1$  plane of the Askey scheme of hypergeometric orthogonal polynomials and are orthogonal on the positive real line, with respect to a discrete weight function. We discuss results on the zeros of these polynomials that were published in three different papers during the past 10 years, which include results on the location of the first few zeros of the quasi-orthogonal order 1 and quasi-orthogonal order 2 Meixner polynomials.

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## *Continuous dual Hahn polynomials and the open KPZ equation*

**Alisa Knizel** (University of Chicago, USA)

14:00–14:45 Friday 14th April Online

In the talk I will discuss how continuous dual Hahn polynomials arise in the study of the stationary measure for the open KPZ equation. Based on joint work with I. Corwin.

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## *Matrix exceptional polynomials: general set-up and Laguerre case*

**Erik Koelink** (Radboud University, Netherlands)

11:45–12:30 Wednesday 12th April Main Lecture Hall

Matrix orthogonal polynomials have been studied in various contexts, and many classical properties of orthogonal polynomials have been extended to matrix orthogonality. Scalar exceptional polynomials are related to a Darboux factorization of a differential (or similar) operator, and the simplest of these factorizations correspond to raising and lowering operators for orthogonal polynomials in the Askey-scheme. We discuss an appropriate analog at the level of a matrix differential operator for matrix orthogonal polynomials and its relation to the corresponding Fourier algebras of operators. We apply this to the special case of matrix Laguerre polynomials, and we discuss some open problems. This is joint work with Lucía Morey and Pablo Román (U Nacional de Córdoba, Argentina).

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## *Planar orthogonal polynomials as multiple orthogonal polynomials*

**Arno Kuijlaars** (University of Leuven, Belgium)

14:00–14:45 Wednesday 12th April Main Lecture Hall

Polynomials with orthogonality in the complex plane play a prominent role in models of non-Hermitian random matrices. I will consider orthogonality weights of the form  $\prod_{j=1}^p |z - a_j|^{2c_j} e^{-|z|^2}$  with complex numbers  $a_j$  and positive  $c_j$ . Lee and Yang [2] showed that the planar orthogonal polynomials are type II multiple orthogonal polynomials on contours. Quite remarkably, in case of integer  $c_j$ , the same polynomials are also type I multiple orthogonal [1]. The two types of multiple orthogonality lead to different Riemann-Hilbert problems, that are potentially useful for asymptotic analysis. I will also discuss work in progress with S. Lahiry and I. Parra in this direction.

## References

- [1] S. Berezin, A.B.J. Kuijlaars and I. Parra, arXiv:2212.06526
  - [2] S.Y. Lee and M. Yang, J. Phys. A. 52 (2019)
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## *Lattice paths, branched continued fractions, and multiple orthogonal polynomials*

**Hélder Lima** (University of Leuven, Belgium)

10:00–10:30    Wednesday 12th April    Main Lecture Hall

The central theme of this talk is the recently found connection between multiple orthogonal polynomials and branched continued fractions. The latter arise as generating functions of lattice paths and were introduced to solve total-positivity problems of combinatorial interest. Production matrices and total positivity play an important role in this connection. I start by giving a brief introduction to the different topics involved in this talk. Then, I present an overview of the general connection between those topics, explaining how the study of the connection between multiple orthogonal polynomials and branched continued fractions brings to light new results on both fields, with emphasis on the results about multiple orthogonal polynomials.

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## *Stokes Phenomenon and Discrete Integrability*

**Christopher Lustrì** (Macquarie University, Australia)

12:00–12:30    Thursday 13th April    Main Lecture Hall

Discrete equations such as the discrete Painlevé I equation can be written in terms of an infinite-order differential equation. We will consider a family of equations obtained by truncating this infinite-order differential equations at different orders. We will study how this system behaves as the differential equation order increases, and how the integrability property of discrete Painlevé I emerges through this analysis. This is joint work with John King (University of Nottingham, UK).

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## *Coherent pair of measures on lattices*

**Dieudonne Mbouna** (University of Porto, Portugal)

10:00–10:30    Friday 14th April    Main Lecture Hall

We study a structure relation between two orthogonal polynomial sequences including their derivatives. This leads to the notion of coherent pair of measures extended to the Askey-Wilson and Wilson operators.

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## *Edge universality of random normal matrices generalizing to higher dimensions*

**Leslie Molag** (University of Sussex, UK)

16:30–17:00    Wednesday 12th April    Main Lecture Hall

The eigenvalues of random normal matrices are described by a correlation kernel, which is constructed out of orthogonal polynomials in the plane. In the large size limit the corresponding eigenvalues accumulate on a two dimensional set (the droplet) in the complex plane. It has recently been proved in generality by Hedenmalm and Wennman that one finds a complementary error function behavior at the boundary of the droplet (the edge). We prove that such universal behaviors transcend this class of random normal matrices, being also valid in higher dimensional determinantal point processes, defined on  $\mathbb{C}^d$ . The models under consideration concern higher dimensional generalizations of the determinantal point processes describing the eigenvalues of the complex Ginibre ensemble and the complex elliptic Ginibre ensemble. In these models, the correlation kernels can be explicitly expressed in terms of certain orthogonal polynomials on  $\mathbb{C}^d$ . Their average density of points converges to a uniform law on a  $2d$ -dimensional hyperellipsoid. It is on the boundary of this region that we find a complementary error function behavior and the Faddeeva plasma kernel. To the best of my knowledge, this is the first instance of the Faddeeva plasma kernel emerging in a higher dimensional model.

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## *Lumer-Phillips on linear relations*

**Boitumelo Moletsane** (University of the Witwatersrand, RSA)

10:00–10:30 Tuesday 11th April Main Lecture Hall

We extend Lumer-Phillips Theorem to relations using the following version: a surjective dissipative operator is  $m$ -dissipative and invertible. This result remains true if dissipative linear relations (i.e. multivalued operators) are considered. The main purpose of the extension is to study  $m$ -dissipative relations. They are very useful for instance when studying the Laplacian on perturbed domains.

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## *Random matrices, orthogonal polynomials and integrable hydrodynamic equations*

**Antonio Moro** (Northumbria University, UK)

14:45–15:30 Wednesday 12th April Main Lecture Hall

We briefly review some key aspects of the well established mutual connections between the theory of random matrix ensembles, orthogonal polynomials and integrable systems. On this basis, we discuss how elements of emergent complexity in random matrix theory and orthogonal polynomials can be understood in terms of multiscale asymptotics and normal forms of integrable nonlinear PDEs

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## *Asymptotic behaviour of quartiles in the gamma distribution*

**Henrik Pedersen** (University of Copenhagen, Denmark)

11:45–12:30 Tuesday 11th April Main Lecture Hall

The  $p$ -quartile of the gamma distribution with shape parameter  $x$  is an implicitly defined function  $m_p$  of  $x$ . We investigate the asymptotic behavior of  $m_p$  for small, and large, values of the shape parameter, thereby generalizing earlier work concerning the median  $m_{1/2}$ .

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## *Dispersive quantisation, strong and weak*

**Beatrice Pelloni** (Heriot-Watt University, UK)

11:00–11:45 Friday 14th April Online

I will discuss a surprising phenomenon, first observed experimentally in linear optics and quantum wave transmission, and known variously as the Talbot effect, fractalisation, (quantum) revivals, or dispersive quantisation. This phenomenon is manifested in the solution of periodic dispersive equation, starting for a discontinuous initial condition. Then, at times that are rational multiple of the spatial period (“rational” times), the solution is discontinuous, indeed it is built from translated copies of the initial condition, while at all other times, the solution is wildly oscillatory, with positive fractal dimension, but it is continuous, so that the solution has more regularity than the initial state. I will discuss the mathematical description of this phenomenon, and the effect on it of nonlinearity, integrability and non-periodic boundary conditions.

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## *Racah algebras and superintegrable systems*

**Sarah Post** (University of Hawai’i at Mānoa, USA)

9:00–9:45 Thursday 13th April Online

In this talk, I will give a brief survey of Racah algebras and their connection to superintegrable systems. The Racah algebra was first encountered looking at integrals of motion for systems in  $2D$  and these systems have a natural generalization to  $nD$ . However, complications arise when considering these higher rank extensions more abstractly. I will discuss some of these complications and recent advances in studying the representation theory of the higher rank cases.

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## *The direct problem for a large class of orthogonal polynomials on the real line*

**Maria das Neves Rebocho** (Universidade da Beira Interior, Portugal)

14:45–15:30 Friday 14th April Online

The construction of families of orthogonal polynomials on the real line from a given modified orthogonalizing measure has been subject of many studies. Well-known connections with integrable systems, Painlevé equations (discrete and continuous), random matrices, and many other topics from the literature of Mathematical-Physics have been recently studied. In this talk we focus on the so-called Laguerre-Hahn class on the real line, that is, the sequences of orthogonal polynomials whose Stieltjes functions satisfy a Riccati type differential equation with polynomial coefficients. The main goal is to study the so-called direct problem for Laguerre-Hahn orthogonal polynomials: to deduce properties of the recurrence coefficients of the orthogonal polynomials from the knowledge of the polynomials involved in the Riccati equation. We shall take Stieltjes functions subject to a deformation parameter,  $t$ , and we derive systems of differential equations and give Lax pairs, yielding non-linear differential equations in  $t$  for the recurrence relation coefficients and Lax matrices of the orthogonal polynomials.

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## *The inverse problem associated with the two-sided Ellis-Gohberg identities for orthogonal Wiener-class functions*

**Sanne Ter Horst** (North-West University, RSA)

11:00–11:45 Saturday 15th April Main Lecture Hall

In 1955 M.G. Krein considered the analogue of the Szegő orthogonal polynomials in the context of convolution operators on finite intervals, as well as the corresponding inverse problem, in which one wants to reconstruct the weight function associated with a given orthogonal polynomial, and together with H. Langer he extended the results on the location of zeros for such functions. These results, as well as the original results of Szegő, were further extended by R.L. Ellis and I. Gohberg (and D.C. Lay) to sequences of orthogonal Wiener-class functions. In this talk we focus on the extension of some of these results to matrix-valued Wiener functions, more specifically, I will mainly focus on the inverse problem associated with the two-sided Ellis-Gohberg identities.

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## *A finite sequence of hypergeometric discrete orthogonal polynomials with four free parameters extending the Hahn tableau*

**Daniel Tcheutia** (AIMS, Cameroon)

11:00–11:45 Wednesday 12th April Main Lecture Hall

By considering the specific Sturm-Liouville problem

$$\sigma(x)\Delta\nabla y(x) + \tau(x)\Delta y(x) + \lambda y(x) = 0,$$

where

$$\Delta y(x) = \nabla y(x+1) = y(x+1) - y(x),$$

and  $\sigma(x) + \tau(x) = (x+p)^2 + q^2$ ,  $\sigma(x+1) = (x+r)^2 + s^2$ , we introduce a new finite sequence of Hahn-type discrete polynomials with four free parameters and prove that they are finitely orthogonal on the real line. We then compute their norm square value by using Dougall's bilateral sum and obtain all moments corresponding to the introduced polynomials. Some structure relations of this polynomial family are given, from which the inversion and the connection formulae of the family are derived.

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## *Discrete Painlevé I and II for orthogonal polynomials*

**Walter Van Assche** (University of Leuven, Belgium)

9:45–10:30 Thursday 13th April Main Lecture Hall

We will explain that the recurrence coefficients of the orthogonal polynomials on the real line for the weight function  $w(x) = e^{-x^4+tx^2}$  satisfy a non-linear recurrence relation which can be identified as discrete Painlevé I, which is  $d\text{-P}(A_2^{(1)}/E_6^{(1)})$  in contemporary notation. They also satisfy a non-linear differential equation in  $t$  which is Painlevé IV for a special choice of parameters. The required solution is a special solution in terms of parabolic cylinder functions and for the discrete Painlevé I equation this is the unique solution with  $x_0 = 0$  for which  $x_n > 0$  for all  $n > 0$ . We also show that the recurrence coefficients of the orthogonal polynomials on the unit circle for the weight  $w(\theta) = e^{t \cos \theta}$  satisfy a non-linear recurrence relation which corresponds to discrete Painlevé II, which is  $d\text{-P}(A_3^{(1)}/D_5^{(1)})$  in contemporary notation. The ratio of two consecutive recurrence coefficients also satisfies a differential equation in  $t$  which is Painlevé III. One needs the special solution in terms of modified Bessel functions. For the discrete Painlevé II equation one needs the solution with  $x_0 = -1$  for which  $-1 < x_n < 1$  for all  $n > 0$ . We show that the connection between the discrete Painlevé equations and the Painlevé differential equations is given by the Toda lattice equations or related lattice equations.

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## *Realization formulas for involutions of matrix-valued positive real odd functions*

**Alma van der Merwe** (University of the Witwatersrand, RSA)

16:00–16:30 Tuesday 11th April Main Lecture Hall

In a seminal paper Foster showed that the impedances of lumped electrical circuits generated by inductances and capacitors are positive real odd functions (PRO for short). For multi-port electrical systems built from inductances and capacitors one obtains matrix-valued PRO functions, denoted  $\text{PRO}_m$  in the case of  $m \times m$  matrix functions. Like PRO, the class of matrix functions  $\text{PRO}_m$  is also a convex invertible cone, i.e., a convex cone closed under inversion (in the form of involution). Given a minimal, Weierstrass descriptor realization for a function in  $\text{PRO}_m$ , we explicitly compute a minimal, Weierstrass descriptor realization for its involution, and through these formulas one can analyse the zero-pole structure of the function.

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## *The Pole Field Solver and the Painlevé Equations*

**André Weideman** (Stellenbosch University, RSA)

10:00–10:30 Saturday 15th April Main Lecture Hall

The Painlevé equations pose a significant number of challenges to numerical computation, not least of which is the typical occurrence of dense pole fields throughout the complex plane. A dozen years ago, Bengt Fornberg and the author introduced the pole field solver, an algorithm that enables one to traverse large sections of such pole fields in a numerically accurate and stable way. The key ingredients are a pole friendly time stepping method (based on Padé approximation) combined with a path selection strategy that preserves numerical stability. In this talk we review the details of the algorithm, and how it was used to catalog several classes of solutions in the Painlevé family.

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# Participants

**Mohamed Jalel Atia** (Qassim University, Saudi Arabia)

**Sung-Soo Byun** (Korea Institute for Advanced Study, Republic of Korea)

**Yves Guemo Tefo** (University of Dschang, Cameroon)

**Priyabrat Gochhayat** (Sambalpur Univeristy, India)

**Kerstin Jordaan** (University of South Africa, RSA)

**Salifou Mboutngam** (University of Maroua, Cameroon)

**Ben Mitchell** (University of Kent, UK)

**Pinaki Kar** (Sambalpur University, India)

**Patrick Njionou Sadjang** (University of Douala, Cameroon)

**Shivani Singh** (University of South Africa, RSA)

**Kaobaiba Wakreo** (University of Maroua, Cameroon)

**Meng Yang** (University of Copenhagen, Denmark)

**Jaccie Zeelie** (North-West University, RSA)