T4T 2022

Topology for Tomorrow

A workshop for the new generation of South African topologists, held from 2 – 5 December 2022 at the African Institute for Mathematical Sciences (AIMS) in Muizenberg, Cape Town. The workshop is a satellite event of the annual SAMS Congress and aims to contribute to building a cohort of future topology researchers who will work together and build further international partnerships.

The daily programme consists of three lecture series given by our guest speakers:

- **A PRIMER ON POINTFREE TOPOLOGY**
  Prof Jorge Picado, *CMUC, Department of Mathematics, University of Coimbra, PORTUGAL*
- **SEMILATTICES AND RELATED CATEGORIES**
  Prof James J. Madden, *Department of Mathematics, Louisiana State University, USA*
- **COHOMOLOGY OF CATEGORIES AND CATEGORIFICATION OF COHOMOLOGY**
  Prof Frank Neumann, *School of Computing and Mathematical Sciences, University of Leicester, UK*

Each day will end with a problem session where workshop participants will work on problems shared by the lecturers, some to assist in deepening understanding of the lecture material and some leading towards new research direction.

On the final day there will be a panel discussion on the topic of Topological Data analysis (TDA) asking what mathematics lies behind it and examining its relation to topology research in South Africa.

Daily Programme is given on the last page. There will be two **Social Events**, a welcome reception at Casa Labia on Friday 2 December and a dinner at the Brass Bell restaurant on Sunday 4 December. Lunches will be taken each day at AIMS and those staying in Muizenberg will take their breakfasts as well as dinner on Saturday 3 December at AIMS as well.

The workshop has been made possible thanks to the generous financial support of a number of universities and organisations. We encourage all participants to respect this investment and take part fully in all the workshop activities.

ABSTRACTS OF LECTURE SERIES

A PRIMER ON POINTFREE TOPOLOGY

JORGE PICADO
CMUC, Department of Mathematics, University of Coimbra, PORTUGAL

Pointfree topology is an approach to general topology developed in the category **Loc** of locales and localic maps ([1, 2]). In some respects, **Loc** is nicer than its classical counterpart **Top** (of topological spaces and continuous maps): it contains many objects that are not space-like, it can be analysed by algebraic methods, and it provides tools that are not so obvious from a classical perspective.

In these lectures, we will present a brief overview of the ideas and motivation for pointfree thinking, with some illustrative examples. We plan to cover the following topics, explaining the similarities and dissimilarities with the classical setting and emphasizing the new features:

- The category **Frm** (of frames and frame homomorphisms) and its dual category **Loc** (of locales and localic maps). Galois adjunctions. Frames as complete Heyting algebras.
- The "starting point of pointfree topology": the contravariant functor O: **Top** $\to$ **Frm** and the adjoint functor Ω: **Top** $\to$ **Loc** .
- Locales as generalised spaces. Sublocales of a locale. Images and preimages. The "fundamental picture of pointfree topology".
- Points of a locale. The spectrum space of a locale.
- The relation between subspaces and sublocales of a spatial locale $\Omega(X)$. Some nice consequences: an outline of the advantages and merits of a pointfree approach to measure theory ([3]).

References

[1] J. Picado and A. Pultr, Frames and locales: Topology without points , Frontiers in Mathematics, vol. 28, Springer, Basel (2012).
[2] J. Picado and A. Pultr, Separation in point-free topology , Birkh¨auser/Springer, Cham (2021).
[3] A. Simpson, Measure, randomness and sublocales , Annals of Pure and Applied Logic 163 (2012) 1642–1659.

SEMILATTICES AND RELATED CATEGORIES

JAMES J. MADDEN
Department of Mathematics, Louisiana State University, USA

A *semilattice* is a set equipped with an associative, commutative, idempotent operation with identity, i.e., an idempotent commutative monoid. For example, a poset with a smallest element in which any two elements have a least upper bound is a semilattice under the least-upper-bound operation; such a poset is called a *join-semilattice*. A fundamental theorem states that the category of join-semilattices is equivalent to the category of algebraic lattices with compactness-preserving complete join-homomorphisms. I will describe the proof in detail because it shows very nicely what `equivalence' means in category theory and how an equivalence of categories is proved. This theorem has many applications in frame theory, which I will describe. Following this, I will explain

``Pontryagin Duality'' for semilattices and present an elementary proof. Time permitting, I will discuss some recent work on conjunctive join-semilattices.

I plan to include comments on why semilattices are worthy of study and where they fit in the broader mathematical landscape. Throughout these talks, I will be mentioning accessible research questions that require little more background than what I will be presenting (but probably much chewing). Finally, I will supply a guide to the recent literature on the topics closely related to what I will be talking about.

COHOMOLOGY OF CATEGORIES AND CATEGORIFICATION OF COHOMOLOGY

FRANK NEUMANN
School of Computing and Mathematical Sciences, University of Leicester, UK

In these lectures I will discuss various (co)homology theories for categories new and old due to Baues-Wirsching, Hochschild-Mitchell, Watts, Roos, Quillen and others and how to relate, unify and generalize them using techniques from simplicial homotopy theory originally due to Gabriel-Zisman and Thomason. We will derive their main homological properties and develop calculational tools in the form of spectral sequences. We will also outline some interesting applications to algebraic topology, algebraic geometry and number theory. Furthermore, we will indicate how to categorify these cohomology theories to obtain cohomology theories for 2-categories (and higher) and how this is related to the theory of decomposition spaces and their combinatorics.

The lectures are based on a series of articles and work in progress with A. Tonks (Malaga) and I. Gálvez-Carillo (UPC Barcelona).

References

[1] F. Neumann, T. Pirashvili, A. Tonks: Cohomology of stacks with values in Picard categories, in preparation.
[2] I. Gálvez Carrillo, F. Neumann, S. Paoli, A. Tonks: Cohomology of higher categories. in preparation.
[3] B. Richter: From Categories to Homotopy Theory, Cambridge Studies in Advanced Mathematics Series Vol. 188, Cambridge University Press, 2020.
[4] I. Gálvez-Carrillo, F. Neumann, A. Tonks: Gabriel-Zisman cohomology and spectral sequences. Applied Categorical Structures 29 (2021), 69-94.
[5] I. Gálvez-Carrillo, F. Neumann, A. Tonks: Thomason cohomology of categories. Journal of Pure and Applied Algebra 217 (2013), 2163-2179.
[6] I. Galvez-Carrillo, F. Neumann, A. Tonks: André spectral sequences for Baues-Wirsching cohomology of categories, Journal of Pure and Applied Algebra 216 (2012), 2549-256.
[7] H. J. Baues, G. Wirsching, Cohomology of small categories, J. Pure Appl. Algebra 38 (1985), 187-211.
[8] D. Quillen, Higher algebraic K-theory. I. Algebraic K-theory, I: Higher K-theories (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972), pp. 85--147. Lecture Notes in Math. 341, Springer, Berlin 1973.
[9] B. Mitchell, Rings with several objects, Advances in Math. 8 (1972), 1-161.
[10] M. André, Limites et fibrés, C. R. Acad. Sci. Paris, Sér. A. 260 (1965), 756-759

T4T 2022 WORKSHOP PROGRAMME

Welcome function: 18:00 on Friday 2 December at Casa Labia (192 Main Rd, Muizenberg)

Saturday 3 December			**Sunday 4 December**			**Monday 5 December**		
Time	**Activity**		**Time**	**Activity**		**Time**	**Activity**	
08:00 -	Breakfast		08:00 -	Breakfast		08:00 -	Breakfast	
08:30 - 10:30	Opening **Pointfree Topology**	Jorge Picado	09:00 - 10:30	**Pointfree Topology**	Jorge Picado	09:00 - 10:30	**Pointfree Topology / Semilattices**	Jorge Picado & James Madden
10:30 - 11:00	Coffee break		10:30 - 11:00	Coffee break		10:30 - 10:45	Coffee break	
11:00 - 12:30	**Semilattices**	James Madden	11:00 - 12:30	**Semilattices**	James Madden	10:45 - 12:30	**Is there any topology in TDA?** Panel discussion with panellists: Vivien Visaya, Jeff Sanders, Loyiso Nongxa and Frank Neumann Chair: David Holgate Closing	
12:30 - 13:30	Lunch		12:30 - 13:30	Lunch		12:30 - 13:30	Lunch	
13:30 - 15:00	**Cohomology of categories**	Frank Neumann	13:30 - 15:00	**Cohomology of categories**	Frank Neumann	13:30	Depart for SAMS Congress	
15:00 - 15:30	Coffee break		15:00 - 15:30	Coffee break	Group Photograph			
15:30 - 16:30	**Problem solving session**		15:30 - 16:30	**Problem solving session**				
18h00 -	Supper at AIMS for delegates staying in Muizenberg		18:30 -	Supper at the Brass Bell, Kalk Bay (Depart from AIMS at 18:00)				