**Structured MSc in Mathematical Sciences -   January 2021 intake**

**Courses overview and abstracts**

| Dates | Course | Lecturer | Affiliation |
|---|---|---|---|
| 22-26 February 2021 | Introduction to Computing and LaTeX | Jan Groenewald | AIMS South Africa |
| 1-19 March 2021 | Python Programming | Paul Taylor Martha Kamkuemah | National Institutes of Health Stellenbosch University |
| | The Geometry of Maxwell's Equations | Tevian Dray Corinne Manogue | Oregon State University |
| | Financial Mathematics | Ronnie Becker Hans-Georg Zimmerman | AIMS South Africa |
| | Data Science | Bubacarr Bah Pete Grindrod Hans-Georg Zimmerman | AIMS South Africa Oxford University Fraunhofer IBMT |
| 22 March to 9 April 2021 | Experimental Mathematics with Sage | Yae Gaba Evans Doe Ocansey | African Centre for Advanced Studies Johannes Keppler University |
| | Introduction to Random Systems, Information Theory, and related topics | Stéphane Ouvry | Université Paris Saclay |
| | Networks | Phil Knight | University of Strathclyde |
| | Introduction to Multiscale Models and their Analysis | Jacek Banasiak | University of Pretoria |
| 12- 30 April 2021 | Analytical Techniques in Mathematical Biology | Lyndsay Kerr | Edinburgh University |
| | Computational Algebra | Wolfram Decker and Gerhard Pfister | TU Kaiserslautern |
| | Fluid Dynamics | Grae Worster Richard Katz | University of Cambridge University of Oxford |
| **3-7 May** | **Reading week** | | |
| 10-28 May 2021 | Probability and Statistics | Daniel Nickelsen | AIMS South Africa |
| | Biophysics at the Microscale | Daisuke Takagi | University of Hawaii at Manoa |
| | Symmetry Analysis of Differential Equations | Masood Khalique Abdul Kara | North-West University University of the Witwatersrand |
| | Distributed Systems | Jeff Sanders | AIMS South Africa |
| 31 May to 18 June 2021 | Algebraic Biology | Matt Macauley | Clemson University |
| | Model Theory and Homogeneous Structures | Dugald MacPherson Gareth Boxall | University of Leeds Stellenbosch University |

| | Risk Management and Economics | Juerg Weber | University of Western Australia |
|---|---|---|---|
| | Concepts and Problem Solving in Physics | David Aschman | University of Cape Town |
| 21-25 June 2021 | Entrepreneurship skills and cases | Karen Hidden | Independent consultant |
| 28 June – 16 July 2021 | Mathematical Problem Solving | Dimbinaina Ralaivaosaona | Stellenbosch University |
| | Curves and Surfaces for Computer Graphics | Laure Gouba | Abdus Salam International Centre for Theoretical Physics (ICTP) |
| 19 July to 6 August | Optimisation | Montaz Ali | University of the Witwatersrand |
| | Programming with Julia | Henri Laurie | University of Cape Town |
| 9-27 August 2021 | Numerical Mathematics | Justin Munyakazi | University of the Western Cape |
| | Machine Learning Applied to Medicine | Alessandro Crimi | AIMS Ghana |

**22-26 February**

**Introduction to Computing and LaTeX**
**Jan Groenewald (AIMS South Africa)**

This course introduces students to the AIMS computing facilities and packages. The course covers an introduction to LaTeX using texmaker, followed by working through the book: http://en.wikibooks.org/wiki/LaTeX, and associated documents such as those from the American Mathematical Society.

**1-19 March 2020**

**Python programming**
**Paul Taylor (National Institutes of Health)**
**Martha Kamkuemah (Stellenbosch University)**

The goal of the course is to have the student feel comfortable with the fundamental building blocks of programming, as well as appreciate coding's conceptual ties with the mathematical sciences. We will cover basic functional programming, both in general principles and with specific, Python-based considerations. We frame the course by building on the student's own mathematical background and on the similarity of notation between math/physics and programming. Major topics include: data types, conditionals; functions; collections (lists, arrays and dictionaries); for and while loops; good programming practice and modular structure; data visualization and plotting; reading from and writing to files; and numerical integration.

**The Geometry of Maxwell's Equations**
**Tevian Dray and Corinne Manogue (Oregon State University)**

Electromagnetism is beautifully described using vector calculus, yet most treatments of vector calculus emphasize algebraic manipulation, rather than the geometric reasoning that underpins Maxwell's equations. This course attempts to bridge that gap, providing a unified review of both electro- and magneto-statics and the underlying vector calculus. Course materials will be taken from the American Physical Society national award-winning Paradigms in Physics program at Oregon State University, including an online text and numerous small group activities.

**Financial Mathematics**
**Ronnie Becker (AIMS South Africa)**
**Hans Georg Zimmerman (Fraunhofer IBMT)**

The course will give an introduction to financial mathematics and will discuss the basic concepts necessary for an understanding of the subject. Topics to be covered include portfolio theory, financial Instruments, risk management, no-arbitrage pricing of assets, asset pricing in the binomial model, elements of stochastic calculus, stochastic differential equations and Monte Carlo methods for solving stochastic differential equations. Numerical methods using the computer platform Python will be employed to do calculations on financial data obtained from the internet.

**Data Science**
**Bubacarr Bah (AIMS South Africa)**
**Pete Grindrod  (Oxford University)**
**Hans Georg Zimmerman (Fraunhofer IBMT)**

Data Science is a new interdisciplinary field which is very popular and has a lot of useful applications with a potential to play a leading role in the on-going digital revolution. The course will introduce students to this new field starting with some mathematical foundations of Data Science and traditional machine learning algorithms in the first week; deep learning in the second week; and case studies and ethics in the final week. By the end of the course students would have been exposed to a representative selection of Data Science approaches that will give them the initial push for some of them to take up careers in Data Science.

**22 March – 9 April 2021**

**Experimental Mathematics with Sage**
**Yae Gaba (African Centre for Advanced Studies)**
**Evans Doe Ocansey (Johannes Kepler University)**
The use of a computer algebra system leads the mathematician that we are to a new approach in problem solving, the experiment. This course illustrates how experiments can help mathematicians solve mathematical problems (mainly proving or disproving mathematical claims). These mathematical experiments require computer assistance and, in our case, we will use the System for Algebra and Geometry Experimentation (SAGE). We give an overview and an introduction to the SAGE computer algebra system and its features, covering many aspects of maths, including algebra, combinatorics, numerical mathematics, number theory, calculus. We will work mainly with Discrete Mathematics problems: Linear Algebra, Number theory and possibly some Graph theory. We will be learning along some programming concepts, revisit datatypes and play around with plotting functions, curves, geometric figures and interactive plots. By the end of the course, we expect the student to get familiar with the SAGE computer algebra system, acquire the skills to prepare his own sage notebooks for various purposes.

**Introduction to Random Systems, Information Theory and Related Topics**
**Stéphane Ouvry (Paris Saclay University)**

This course is an introduction to various random systems, probability theory, Shannon information theory and some related topics, with a special emphasis on their mathematical aspects. In particular I will present selected lectures on
- Probability calculus and the central limit theorem
- Application to random walks on a line and Brownian curves
- Notions of random numbers and pseudo random numbers
- Application to Monte Carlo sampling
- Shannon statistical entropy and information theory
- LZW compression algorithm
- Diaconis riffle shuffle: how to "randomize" a deck of cards?
- Random permutations and application to
  the statistical "curse" problem in sailing boat regattas

**Networks**
**Phil Knight (Strathclyde University)**

One cannot ignore the networks we are part of, that surround us in everyday life. There's our network of family and friends; the transport network; the banking network---it doesn't take much effort to come up with dozens of examples. Network theory aims to provide a mathematical framework for analysis of the huge networks that drive the global economy (directly or indirectly) and this course provides an introduction to the key tools and an opportunity to employ them to gain new insight into complex behaviours and structures in real-world data.

The intimate connection between matrix algebra and graph theory will be highlighted and students will use this connection to develop a practical approach to analysing networks. Python provides an ideal computational environment for large-scale simulation and analysis, in particular for identifying the key members of a network and for uncovering local and global structure that can be hidden by the scale of the data.

**Introduction to multiscale models and their analysis**
**Jacek Banasiak (University of Pretoria)**

Natural processes usually are driven by mechanisms widely differing from each other by the time or space scale at which they operate. However, looking at all such scales simultaneously often is infeasible and costly and provides information which is redundant for particular applications. Hence, there has been a growing interest in providing a more focused description of multiscale processes by aggregating variables in a way that is relevant for a particular purpose, and that preserves the salient features of the dynamics without getting bogged down by unnecessary details. One of the problems that occur here is that the aggregation often changes the significantly the structure of the problem leading to the so-called singularly perturbed problems. Many ad hoc methods to deal with singularly perturbed problems have been devised in the applied sciences. The aim of these lectures is to describe some tools which provide a systematic way of deriving the aggregated equations with coefficients encapsulating the relevant information from the discarded levels of description. Since any approximation is only valid if an estimate of the incurred error is available, the tools we describe allow for proving that the solutions to the original multiscale equations converge to the solution of the limit equation if the relevant parameter converges to its critical value.

The course will cover:
1. Dynamical systems toolbox relevant to the topic.
2. Description of multiscale models in mathematical epidemiology, ecology and population dynamics.
3. The method of asymptotic expansions and illustration on linear models.
4. Nonlinear problems – regular and singular perturbations.
5. Introduction to the Tikhonov theorem with applications.

**12-30 April 2021**

**Fluid Dynamics**
**Grae Worster (University of Cambridge)**
**Richard Katz (University of Oxford)**

Fluids are all around us, from the air we breathe to the oceans that determine our climate and from oil that powers our industries to metals that are cast into machinery.  The study of fluid dynamics requires sophisticated applications of mathematics and the ability to translate physical problems into mathematical language and back again.  The course begins by building a fundamental understanding of viscous fluid flows in the context of unidirectional flows.  In more general, higher dimensional flows, pressure gradients are generated within a fluid to deflect the flow around obstacles rather than the fluid being compressed in front of them, and an understanding of the coupling between momentum and mass conservation through the pressure field is key to the understanding and analysis of fluid motions.  We will use simple experiments to illustrate and motivate our mathematical understanding of fluid flow.  Prerequisite for the course is fluency with differential equations and vector calculus.  No previous knowledge of fluid dynamics will be assumed.

**Analytical Techniques in Mathematical Biology**
**Lyndsay Kerr (University of Edinburgh)**

In this class we will explore how analytical techniques and traditional \pure" mathematics can be applied to real biological problems. Mathematical models are mathematical descriptions of real-life systems. In this class we will examine mathematical models that describe important biological systems. Posing such systems in terms of mathematics often proves to be very useful in allowing us to answer questions of interest, such as: What is the long-term outcome of the system? Are there any factors (known as the model parameters) that could cause the long-term behaviour of the system to change? For example, we will examine models that describe the interaction between two competing species. Here, we will be interested in whether both species can coexist in the long term or whether one of the species will die out. Models describing tumour cell growth will also be analysed, where we are interested in whether or not the tumour cells will die out. In addition, we will study models that describe the spread of an infectious disease through a population. Here we will determine what factors contribute to the outbreak or termination of a disease, we will examine the long-term behaviour of the number of infected individuals and we will investigate how immunisation can prevent or halt a pandemic. We will also learn about how real-life data can be used in combination with infection models to estimate important factors such as the infection rate of a disease. The final topic that we will consider relates to the patterning that is found on the coats of animals. We shall examine models describing a potential mechanism that could cause such patterns to arise and we will learn how these models can be used to produce patterns akin to those found on the coats of animals.

### Computational Algebra
### Wolfram Decker and Gerhard Pfister (TU Kaiserslautern)

Groebner bases and Buchberger's algorithm for ideals and modules will be studied. Applications to commutative algebra, selected problems in singularity theory and algebraic geometry will be given as well as applications to electronics and engineering. The course includes an introduction to the computer algebra system SINGULAR and its programming language.

### 10-28 May 2021

### Probability and statistics
### Daniel Nickelsen (AIMS South Africa)

This course offers an introduction to probability and statistics. The notion of probability will be introduced on an intuitive level, to then convey basic rules of probability. These rules will be practiced with many examples. The law of large numbers and the central limit theorem will form the basis for traditional statistical methods like hypothesis testing, confidence intervals, and regression. Finally, some of the statistical methods will be recast as Bayesian inference. The course will be accompanied by practical examples using the freely available statistical software R.

### Biophysics at the Microscale
### Daisuke Takagi (University of Hawaii at Manoa)

The dynamics of microscopic life forms and bio-inspired machines are active areas of modern interdisciplinary research, for example in soft matter physics and microbiome sciences. The development of mathematical models of their behaviour must adequately account for the key physical effects operating at the microscale. This course will begin with an overview of biophysics across a wide range of length scales from the molecular to the organismal level, demonstrating how the scale modifies the relative importance of viscosity, elasticity, and thermal fluctuations. The dynamics of each single cell will be formulated using stochastic differential equations to simulate random walks and richer dynamics emerging from motility and chemotaxis. The course will then predict the spatial distribution of many cells over time by deriving the governing Fokker-Planck equation, and this will be applied to study reaction-diffusion processes and pattern formation in living systems. Computer lab demonstrations and exercises will complement the classes. Prerequisite is basic probability theory and ordinary differential equations. No prior knowledge of biology or fluid physics is required.

**Symmetry analysis of Differential Equations**
**Khalique Masood (North West University)**
**Abdul Kara (University of the Witwatersrand)**

Symmetry methods for solving differential equations, originally developed by Sophus Lie (1842-1899), unifies many existing ad hoc methods for constructing explicit solutions for differential equations and provide powerful new ways to find analytical solutions. Lie invented the theory of Lie groups when studying symmetries of differential equations. This theory has applications in many areas of mathematics such as algebraic topology, differential geometry, bifurcation theory, control theory, classical mechanics, relativity etc. It also has applications in physics, engineering and other mathematically-based sciences. In this course we shall learn how to find Lie point symmetries of an ordinary differential equation and then use them to find exact solutions of the equation.

**Distributed Systems**
**Jeff Sanders (AIMS South Africa)**

Every day we use the web, email and Google's search engine. All are distributed protocols: algorithms which run on resources which are physically distributed. When programming in Python you've considered only resources in a single location. It's time to meet reality. This course is an introduction to distributed systems and their protocols. It has been designed to teach participants to:
(a) design distributed protocols by thinking locally how to achieve global requirements;
(b) appreciate the relevant concerns of ethics, efficiency, privacy and security; (c) be familiar with contemporary examples like: www; communications protocols; public key cryptography and RSA; bitcoin; google rank; zero-knowledge protocols; choice-coordination; oblivious transfer; and quantum information (or distributed quantum computing) if there is time and sufficient interest.
Distributed systems are studied at the level of design, rather than code, with emphasis on reasoning about how their behaviour achieves global correctness. Assignments accordingly emphasise design and not programming.


31 May – 18 June

**Concepts and Problem Solving in Physics**
**David Aschman (University of Cape Town)**
This course shows that physics describes the real world using the language of mathematics. Problem solving techniques such as changing the point of view, using different reference frames, estimating orders of magnitude, dimensional analysis, and numerical approaches will be used. Examples will be taken from physics of moving objects, electrodynamics, gravity, movement of molecules in gases, and elementary particle physics. Students are required to read, think, discuss, engage, interact, argue, present their ideas verbally, do homework, compute and present their ideas verbally and in writing. Details of the topics covered will be available on the course page.


**Risk Management and Insurance Economics**

**Juerg Weber (University of Western Australia)**

The proposed course provides an introduction to the operation of insurance markets and the design of insurance contracts. These topics would be useful to mathematicians who are seeking a career in the actuarial sciences and the insurance industry. The course, which uses some financial mathematics and calculus, would be based on the textbook by P. Zweifel and R. Eisen, 'Insurance Economics', Berlin and Heidelberg, Springer, 2012.


**Algebraic Biology**

**Matt Macauley (Clemson University)**

Mathematical biology has been transformed over the past 15 years by researchers using novel tools from discrete math and computational algebra to tackle old and new problems. For example, many systems such as gene regulatory networks have been traditionally modelled using differential equations. However, a new popular trend is to use finite dynamical systems such as Boolean networks. In this setting, the local functions and the dynamical system map can be expressed as multivariable polynomials. This opens the door to using the powerful toolbox of computational algebra to attack classic problems in systems biology. In this class,

students will be introduced to this new and exciting field known as "algebraic mathematical biology".

## Model Theory and Homogeneous Structures

**Dugald MacPherson (University of Leeds)**

**Gareth Boxall (Stellenbosch University)**

Model theory is the branch of mathematical logic which applies to mathematics abstract concepts from formal logic, such as: languages, structures (such as graphs and groups), and the meaning of a sentence or formula of a formal language when interpreted in a structure. This course will introduce the main concepts of first order logic including the fundamental theorem which makes first order logic so powerful -- the Compactness Theorem. This will not be proved, but applications will be given.

The course will then describe an abstract notion of isomorphism between two (usually infinite) structures, and hence of automorphism of a structure. This leads to the automorphism group of the structure (the natural tool for working with symmetry). By proving and then applying Fraisse's Amalgamation Theorem, we will show how to construct infinite `homogeneous' structures, which have very rich symmetry. We will exploit this to give surprising examples of automorphism groups, and also give a construction of a beautiful graph with vertex set the natural numbers which has the property that any graph with the same vertex set is with probability 1 isomorphic to it. We will finish with examples and classification results for homogeneous structures.

### *21- 25 June 2021*

### Entrepreneurship skills and cases

### Karen Hidden (Independent consultant)

This course introduces students to the discipline and process of entrepreneurship and new venture planning. Entrepreneurship crosses many disciplines and is a pursuit and a subject domain that is now commonly found in technical areas such as science, engineering, technology and mathematics. In this course we will introduce students to the process of originating a business idea and to creating a business model (an 'engine' for economic growth and sustainability) using a tool called the Business Model Canvas. The course will give students both the knowledge and the competence to consider entrepreneurship as part of a future career path.

### 28 June to 16 July 2021

### Mathematical Problem Solving
### Dimbinaina Ralaivaosaona (Stellenbosch University)
n this course we shall consider a variety of elementary, but challenging, problems in different branches of pure mathematics. Investigations, comparisons of different methods of attack, literature searches, solutions and generalizations of the problems will arise in discussions in class. The objective is for students to learn, by example, different approaches to problem solving and research.

### Curves and Surfaces for Computer Graphics
### Laure Gouba (Abdus Salam International Centre for Theoretical Physics, ICTP)

The course is about all important parametric representations and basic concepts such as curvature, tangent vectors, normal vectors, curve and surface continuity, Cartesian products and simplest curves and surfaces.

19 July to 6 August 2021

### Programming with Julia
### Henri Laurie (University of Cape Town)

The Julia language has a solution to the demand for speed *both* in the solution of numerical problems *as well as* in the writing of code for those solutions. These two demands are usually seen as contradictory, but Julia

has shown that it can reach the computational speed of languages like C and Fortran, while it equals languages like Python and R for speed of development. In fact, enthusiasts say that large projects are quicker to code in Julia than any other language.

This course aims to introduce you to both aspects. The first week is devoted to the basics of Julia and of solving ODEs numerically. In the second week is a showcase of ODE modelling, with examples mostly from biology: we look at the variety of problems and solvers (and will cover only a small part of that enormous variety) and the ease of writing perfomant code in Julia. In week three we choose to look at one or two of the following: computing local and global sensitivities, parameter estimation, bifurcations, uncertainty quantification and machine learning.

## Optimisation
**Montaz Ali (University of the Witwatersrand)**

n this topic I will introduce a number of standard but practical discrete optimization problems. These are the shortest path problem, max-flow problem, minimum cost flow problem, linear and quadratic assignment problem, minimum spanning tree problem, knapsack and quadratic knapsack problem, symmetric and asymmetric traveling salesman problem, facility location, vehicle routing and location routing problem, set covering problem, winner determination problem, the minimum vertex cover problem, graph partitioning problem, max-cut problem, and graph coloring problem. Graph theoretical as well as the mathematical models of these problems will be thoroughly studied. The second part of the lectures include various algorithms. A number of solution techniques such as heuristics algorithm, approximate algorithm, and deterministic algorithms will be presented. These are the genetic algorithm, simulated annealing algorithm, 2-opt/k-opt heuristics, iterative improvement local search, tabu search, and branch and bound. In the third part of the lectures, I will present very large scale optimization in compressive sensing, optimization problems in maximum clique, quasi-clique, and planed clique. Finally, I will introduce game theory and Nash Equilibrium in my pen-ultimate couple of lectures.

9-27 August 2021

## Numerical Mathematics
**Justin Munyakazi (University of the Western Cape)**

Real life situations (with important exceptions of combinatorial optimization, cryptography and gene sequencing) are modelled by continuous mathematics. In the natural sciences, engineering, finance and economics, we often encounter models that employ functions of real variables. These models can be linear or nonlinear and may involve derivatives or integrals or both. In many applications, the models cannot be solved analytically. In this course we present a number of numerical techniques to construct approximate solutions. For most algorithms, the issue of efficiency and accuracy will be discussed.

## Machine Learning Applied to Medicine
**Alex Crimi, AIMS Ghana**

Machine learning is nowadliy used in many fields. One of the original focus was on medical imaging and genetics, though other types of data have been used. This course will introduce the basic principles of medical imaging/neuroimaging methods as applied to human subjects, moreover bioinformatics elements will be also given. Topics also include an introduction to neuroimaging physics and image formation, as well as an overview of different neuroimaging applications, including functional MRI, diffusion tensor imaging, magnetic resonance, positron emission tomography imaging. Often the ultimate goal of medical imaging is the detection of patterns or the classification of case and control groups (as health subjects against people with a brain disease). Therefore, hand-on examples in Python will be carried out on classification for different
brain diseases based on machinelearning/deeplearning algorithms. A previously course hold on introduction to neuroimaging will be converted to be more focused on neuroimaging and neuroscience
https://sites.google.com/a/aims.edu.gh/pattern-recognition/