Structured MSc in Mathematical Sciences - November 2020 intake

Review courses – overview and abstracts

Dates	Course	Lecturer	Affiliation
25 January to 5 February	Distributed Coordination	Simukai Utete	AIMS South Africa
28 January to 5 February	Mathematics in Industry Study Group	Workshop	
8-26 February 2021	Differential Equations	Fernando Pestana da Costa	Aberta University
	Algebraic Methods	Karin-Therese Howell	Stellenbosch University
1-19 March 2021	Data Science	Bubacarr Bah Pete Grindrod Hans-Georg Zimmerman	AIMS South Africa Oxford University Fraunhofer IBMT
	The Geometry of Maxwell's Equations	Tevian Dray Corinne Manogue	Oregon State University
	Financial Mathematics	Ronnie Becker Hans-Georg Zimmerman	AIMS South Africa
22 March to 9 April 2021	Introduction to Random Systems, Information Theory, and related topics	Stéphane Ouvry	Université Paris Sud
	Networks	Phil Knight	University of Strathclyde
	Introduction to Multiscale Models and their Analysis	Jacek Banasiak	University of Pretoria
12-30 April 2021	Analytical Techniques in Mathematical Biology	Lyndsay Kerr	Edinburgh University
	Computational Algebra	Wolfram Decker and Gerhard Pfister	TU Kaiserslautern
	Fluid Dynamics	Grae Worster Richard Katz	University of Cambridge University of Oxford
3-7 May	**Reading week**		
10-28 May 2021	Biophysics at the Microscale	Daisuke Takagi	University of Hawaii at Manoa
	Symmetry Analysis of Differential Equations	Masood Khalique	North-West University
	Distributed Systems	Jeff Sanders	AIMS South Africa
31 May to 18 June 2021	Algebraic Biology	Matt Macauley	Clemson University
	Model Theory and Homogeneous Structures	Dugald MacPherson Gareth Boxall	University of Leeds Stellenbosch University
	Risk Management and Insurance Economics	Juerg Weber	University of Western Australia

25 January to 5 February 2021

Distributed Coordination
Simukai Utete (AIMS South Africa)

The course considers methods for coordinating decisions and actions of intelligent agents. In many areas, distributed agents are being deployed to solve problems in concert; for example, mobile robots can be employed in environmental monitoring. How do such systems manage failures, coordinate completion of a shared task, or decide on a common course of action? In multi-sensor networks, diverse sensor information can be combined using data fusion methods. The combination of decisions and the coordination of actions have parallels to data fusion, but significant differences too. The course will consider problems, and paradoxes, of distributed decision making and of distributed coordination. Areas covered will include decision making via voting, auctions and communication. In the practical part of the course, students will design and implement a distributed coordination mechanism to address a problem presented in the class. Understanding distributed coordination problems is important for realising systems in which individual intelligent agents can work in harmony and with high levels of autonomy.

28 January to 5 February 2021

Mathematics in Industry Study Group (workshop)

Review Courses

8-26 February

Differential Equations
Fernando Pestana da Costa (Aberta University)

The goal of this course is to be an introduction to the theory and applications of Ordinary Differential Equations, with emphasis in methods of qualitative theory. It starts by revisiting basic techniques to solve ODEs: separable equations, integrating factors, and changes of variables. After these preliminaries the general theorem of Picard-Lindelof for existence and uniqueness of solutions to initial value problems, as well as results on dependence of solutions on the initial data and parameters are studied. These preliminary general results are followed by the main part of the course, viz. a study of important tools from qualitative theory. These consist in the introduction of the main concepts (phase space, flows, critical points, orbits, conservative systems, first integrals, phase portraits, etc.), notions and results about stability of solutions, Lyapunov functions, limit sets, and a study of planar autonomous systems. The final part of the course deals with linear systems (including the computation of the matrix exponential) and the study of the linearization method for nonlinear systems (including the notion of conjugation, and the statement ---without proofs--- and use of the Hartman-Grobman and Hadamard-Perron theorems.) All topics are motivated and exemplified by adequate examples. The discussion and resolution of exercises and problems illustrating the notions, results, and methods studied is an integral part of the course.

Algebraic Methods
Karin-Therese Howell (Stellenbosch University)

The course will have an Algebra and a Linear Algebra component. Under Algebra we will discuss functions, relations, partitions, groups, morphisms, quotients, the Isomorphism Theorems for groups and direct products. As part of the Linear Algebra component we will discuss linear spaces, linear functionals and operators, matrices, change of basis, eigenvalues and eigenvectors and the normal form.

1-19 March 2020

The Geometry of Maxwell's Equations
Tevian Dray and Corinne Manogue (Oregon State University)

Electromagnetism is beautifully described using vector calculus, yet most treatments of vector calculus emphasize algebraic manipulation, rather than the geometric reasoning that underpins Maxwell's equations. This course attempts to bridge that gap, providing a unified review of both electro- and magneto-statics and the underlying vector calculus. Course materials will be taken from the American Physical Society national award-winning Paradigms in Physics program at Oregon State University, including an online text and numerous small group activities.

Financial Mathematics
Ronnie Becker (AIMS South Africa)
Hans Georg Zimmerman (Fraunhofer IBMT)

The course will give an introduction to financial mathematics and will discuss the basic concepts necessary for an understanding of the subject. Topics to be covered include portfolio theory, financial Instruments, risk management, no-arbitrage pricing of assets, asset pricing in the binomial model, elements of stochastic calculus, stochastic differential equations and Monte Carlo methods for solving stochastic differential equations. Numerical methods using the computer platform Python will be employed to do calculations on financial data obtained from the internet.

Data Science
Bubacarr Bah (AIMS South Africa)
Pete Grindrod (Oxford University)
Hans Georg Zimmerman (Fraunhofer IBMT)

Abstract to be supplied.

22 March – 9 April 2021

Introduction to Random Systems, Information Theory and Related Topics
Stéphane Ouvry (Paris-Sud University)

This course is an introduction to various random systems, probability theory, Shannon information theory and some related topics, with a special emphasis on their mathematical aspects. In particular I will present selected lectures on

- Probability calculus and the central limit theorem
- Application to random walks on a line and Brownian curves
- Notions of random numbers and pseudo random numbers
- Application to Monte Carlo sampling
- Shannon statistical entropy and information theory
- LZW compression algorithm
- Diaconis riffle shuffle: how to “randomize” a deck of cards?
- Random permutations and application to the statistical “curse” problem in sailing boat regattas

Networks
Phil Knight (Strathclyde University)

One cannot ignore the networks we are part of, that surround us in everyday life. There's our network of family and friends; the transport network; the banking network---it doesn't take much effort to come up with dozens of examples. Network theory aims to provide a mathematical framework for analysis of the huge networks that drive the global economy (directly or indirectly) and this course provides an introduction to the key tools and an opportunity to employ them to gain new insight into complex behaviours and structures in real-world data.

The intimate connection between matrix algebra and graph theory will be highlighted and students will use this connection to develop a practical approach to analysing networks. Python provides an ideal computational environment for large-scale simulation and analysis, in particular for identifying the key members of a network and for uncovering local and global structure that can be hidden by the scale of the data.

Introduction to multiscale models and their analysis
Jacek Banasiak (University of Pretoria)

Natural processes usually are driven by mechanisms widely differing from each other by the time or space scale at which they operate. However, looking at all such scales simultaneously often is infeasible and costly and provides information which is redundant for particular applications. Hence, there has been a growing interest in providing a more focused description of multiscale processes by aggregating variables in a way that is relevant for a particular purpose, and that preserves the salient features of the dynamics without getting bogged down by unnecessary details. One of the problems that occur here is that the aggregation often changes the significantly the structure of the problem leading to the so-called singularly perturbed problems. Many ad hoc methods to deal with singularly perturbed problems have been devised in the applied sciences. The aim of these lectures is to describe some tools which provide a systematic way of deriving the aggregated equations with coefficients encapsulating the relevant information from the discarded levels of description. Since any approximation is only valid if an estimate of the incurred error is available, the tools we describe allow for proving that the solutions to the original multiscale equations converge to the solution of the limit equation if the relevant parameter converges to its critical value.

The course will cover:

1. Dynamical systems toolbox relevant to the topic.
2. Description of multiscale models in mathematical epidemiology, ecology and population dynamics.
3. The method of asymptotic expansions and illustration on linear models.
4. Nonlinear problems – regular and singular perturbations.
5. Introduction to the Tikhonov theorem with applications.

12-30 April 2021

Fluid Dynamics
Grae Worster (University of Cambridge)
Richard Katz (University of Oxford)

Fluids are all around us, from the air we breathe to the oceans that determine our climate and from oil that powers our industries to metals that are cast into machinery. The study of fluid dynamics requires sophisticated applications of mathematics and the ability to translate physical problems into mathematical language and back again. The course begins by building a fundamental understanding of viscous fluid flows in the context of unidirectional flows. In more general, higher dimensional flows, pressure gradients are generated within a fluid to deflect the flow around obstacles rather than the fluid being compressed in front of them, and an understanding of the coupling between momentum and mass conservation through the pressure field is key to the understanding and analysis of fluid motions. We will use simple experiments to illustrate and motivate our mathematical understanding of fluid flow. Prerequisite for the course is fluency with differential equations and vector calculus. No previous knowledge of fluid dynamics will be assumed.

Analytical Techniques in Mathematical Biology
Lyndsay Kerr (University of Edinburgh)

In this class we will explore how analytical techniques and traditional “pure" mathematics can be applied to real biological problems. Mathematical models are mathematical descriptions of real-life systems. In this class we will examine mathematical models that describe important biological systems. Posing such systems in terms of mathematics often proves to be very useful in allowing us to answer questions of interest, such as: What is the long-term outcome of the system? Are there any factors (known as the model parameters) that could cause the long-term behaviour of the system to change? For example, we will examine models that describe the interaction between two competing species. Here, we will be interested in whether both species can coexist in the long term or whether one of the species will die out. Models describing tumour cell growth will also be analysed, where we are interested in whether or not the tumour cells will die out. In addition, we will study models that describe the spread of an infectious disease through a population. Here we will determine what factors contribute to the outbreak or termination of a disease, we will examine the long-term behaviour of the number of infected individuals and we will investigate how immunisation can prevent or halt a pandemic. We will also learn about how real-life data can be used in combination with infection models to estimate important factors such as the infection rate of a disease. The final topic that we will consider relates to the patterning that is found on the coats of animals. We shall examine models describing a potential mechanism that could cause such patterns to arise and we will learn how these models can be used to produce patterns akin to those found on the coats of animals.

Computational Algebra
Wolfram Decker and Gerhard Pfister (TU Kaiserslautern)

Groebner bases and Buchberger's algorithm for ideals and modules will be studied. Applications to commutative algebra, selected problems in singularity theory and algebraic geometry will be given as well as applications to electronics and engineering. The course includes an introduction to the computer algebra system SINGULAR and its programming language.

10-28 May 2021

Biophysics at the Microscale
Daisuke Takagi (University of Hawaii at Manoa)

The dynamics of microscopic life forms and bio-inspired machines are active areas of modern interdisciplinary research, for example in soft matter physics and microbiome sciences. The development of mathematical models of their behaviour must adequately account for the key physical effects operating at the microscale. This course will begin with an overview of biophysics across a wide range of length scales from the molecular to the organismal level, demonstrating how the scale modifies the relative importance of viscosity, elasticity, and thermal fluctuations. The dynamics of each single cell will be formulated using stochastic differential equations to simulate random walks and richer dynamics emerging from motility and chemotaxis. The course will then predict the spatial distribution of many cells over time by deriving the governing Fokker-Planck equation, and this will be applied to study reaction-diffusion processes and pattern formation in living systems. Computer lab demonstrations and exercises will complement the classes. Prerequisite is basic probability theory and ordinary differential equations. No prior knowledge of biology or fluid physics is required.

Symmetry analysis of Differential Equations
Khalique Masood (North West University)

Symmetry methods for solving differential equations, originally developed by Sophus Lie (1842-1899), unifies many existing ad hoc methods for constructing explicit solutions for differential equations and provide powerful new ways to find analytical solutions. Lie invented the theory of Lie groups when studying symmetries of differential equations. This theory has applications in many areas of mathematics such as algebraic topology, differential geometry, bifurcation theory, control theory, classical mechanics, relativity etc. It also has applications in physics, engineering and other mathematically-based sciences. In this course we shall learn how to find Lie point symmetries of an ordinary differential equation and then use them to find exact solutions of the equation.

Distributed Systems
Jeff Sanders (AIMS South Africa)

Every day we use the web, email and Google's search engine. All are distributed protocols: algorithms which run on resources which are physically distributed. When programming in Python you've considered only resources in a single location. It's time to meet reality. This course is an introduction to distributed systems and their protocols. It has been designed to teach participants to:
(a) design distributed protocols by thinking locally how to achieve global requirements;
(b) appreciate the relevant concerns of ethics, efficiency, privacy and security; (c) be familiar with contemporary examples like: www; communications protocols; public key cryptography and RSA; bitcoin; google rank; zero-knowledge protocols; choice-coordination; oblivious transfer; and quantum information (or

distributed quantum computing) if there is time and sufficient interest.
Distributed systems are studied at the level of design, rather than code, with emphasis on reasoning about how their behaviour achieves global correctness. Assignments accordingly emphasise design and not programming.

31 May – 18 June

Risk Management and Insurance Economics

Juerg Weber (University of Western Australia)

The proposed course provides an introduction to the operation of insurance markets and the design of insurance contracts. These topics would be useful to mathematicians who are seeking a career in the actuarial sciences and the insurance industry. The course, which uses some financial mathematics and calculus, would be based on the textbook by P. Zweifel and R. Eisen, 'Insurance Economics', Berlin and Heidelberg, Springer, 2012.

Algebraic Biology

Matt Macauley (Clemson University)

Mathematical biology has been transformed over the past 15 years by researchers using novel tools from discrete math and computational algebra to tackle old and new problems. For example, many systems such as gene regulatory networks have been traditionally modelled using differential equations. However, a new popular trend is to use finite dynamical systems such as Boolean networks. In this setting, the local functions and the dynamical system map can be expressed as multivariable polynomials. This opens the door to using the powerful toolbox of computational algebra to attack classic problems in systems biology. In this class, students will be introduced to this new and exciting field known as "algebraic mathematical biology".

Model Theory and Homogeneous Structures

Dugald MacPherson (University of Leeds)

Gareth Boxall (Stellenbosch University)

Model theory is the branch of mathematical logic which applies to mathematics abstract concepts from formal logic, such as: languages, structures (such as graphs and groups), and the meaning of a sentence or formula of a formal language when interpreted in a structure. This course will introduce the main concepts of first order logic including the fundamental theorem which makes first order logic so powerful -- the Compactness Theorem. This will not be proved, but applications will be given.

The course will then describe an abstract notion of isomorphism between two (usually infinite) structures, and hence of automorphism of a structure. This leads to the automorphism group of the structure (the natural tool for working with symmetry). By proving and then applying Fraisse's Amalgamation Theorem, we will show how to construct infinite `homogeneous' structures, which have very rich symmetry. We will exploit this to give surprising examples of automorphism groups, and also give a construction of a beautiful graph with vertex set the natural numbers which has the property that any graph with the same vertex set is with probability 1 isomorphic to it. We will finish with examples and classification results for homogeneous structures.